Household Sector Financial Frictions in Canada*

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Abstract

We construct a small open economy model with financial frictions in the household sector. In an attempt to match aggregate dynamics, the model includes several real and nominal frictions. We find that financial frictions play an important role in allowing the model to match several key stylized facts.

1 Introduction

In recent years house prices have increased rapidly across a broad spectrum of countries. In Canada, over the past two decades, the volatility of house price inflation has been 4 times the volatility of consumer price inflation generally. These developments have led to a sustained and active debate on the appropriate role for house prices in the formulation of monetary policy. Similar concerns have arisen in a number of countries. In recent comments to the *Financial Times*, Mervyn King, the governor of the Bank of England, said, "house prices...have been very important to our judgments, because they have represented some of the most important pieces of news." In this paper, we attempt to construct a model that includes a role for house prices in the monetary transmission mechanism and will eventually be useful for policy analysis. This line of work is consistent with an emerging trend at central banks to use DSGE models for projection and policy analysis.[1] Our approach extends this previous work by explicitly incorporating financial frictions.[2] Importantly, we show that

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[1] Examples of this include the SIGMA model at the Federal Reserve Board, the TOTEM model at the Bank of Canada and Smet-Wouters model at the ECB. For details on SIGMA see Erceg, Guerrieri and Gust (2004), on TOTEM see Binette et al. (2004), and on the Smets-Wouters model see Smets and Wouters (2003).

[2] This paper is part of an ongoing project at the Bank of Canada to develop a model for policy analysis that includes financial frictions in several sectors. Although, the current paper

our model is able to replicate certain key stylized facts about housing and the macroeconomy.

Our focus on housing is motivated, in part, by its sheer size on household sector balance sheets. In Canada, as in most industrialized countries, housing is the largest component of household wealth. The aggregate value of housing is roughly double the value of equity owned by households. In addition, more than two-thirds of household credit is secured by housing.

Other authors have considered the role of house prices in the monetary transmission mechanism. Aoki, Proudman and Vlieghe (2002) adapt the Bernanke, Gertler and Gilchrist (1999) (BGG) model to the household sector. They show that the introduction of BGG-style financial frictions amplifies and propagates the effects of monetary policy shocks on residential investment, house prices and consumption. Iacoviello (2005) introduces a borrowing constraint into an otherwise standard sticky-price business cycle model. His estimates indicate that the borrowing constraint allows the model to fit the data somewhat better. However, these papers have studied household sector financial frictions in environments that ignore several other potentially important frictions.

The empirical DSGE literature has found a number of shocks and frictions to be important for explaining business cycle dynamics. For example, Christiano, Eichenbaum and Evans (2005) (CEE) find important roles for habit persistence, investment adjustment costs, variable capacity utilization and dynamically indexed sticky wages and prices. Smet and Wouters (2003, 2004) find a number of different shocks to be important for explaining various dimensions of aggregate fluctuations.[3] This literature has incorporated a wide variety of frictions, but has yet to study household sector financial frictions.

The goal of this paper is to bring together these two strands of the literature by introducing household sector financial frictions into a model with a number of other shocks and frictions. We assess the marginal importance of household sector financial frictions when other important frictions are included in the model. We undertake this analysis in a model that includes the frictions identified above, plus open economy features that we believe are essential for understanding the Canadian economy. Our results suggest that the incorporation of household sector financial frictions improves the fit of the model.

Following Iacoviello (2005), we model financial frictions by assuming that some households face a binding borrowing constraint. The amount they can borrow is limited to a fraction of their housing wealth, thus introducing a financial accelerator mechanism. The household sector financial accelerator operates through a different mechanism but has similar effects to the business sector financial accelerator described by BGG. Suppose some shock causes an increase in house prices, *ceteris paribus*. This would make it possible for borrowing constrained households to borrow more. These households spend their additional

presents a version of the model that includes financial frictions only in the household sector, the complete model will include business sector financial frictions as well. The eventual goal is to be able to analyze the role of financial mechanisms in macroeconomic developments.

[3] Christiano, Motto and Rostagno (2003) also find a role for financial shocks and frictions in the business sector in explaining the Great Depression.

funds on consumption and residential investment. This, in turn, puts upward pressure inflation and house prices. The borrowing constraint eases commensurately with the rising house prices, further reinforcing the effects of the initial shock.

We show that the incorporation of financial frictions related to housing allows us to generate high house price volatility and a positive correlation between consumption and house prices, as observed in the Canadian data. Other authors have found it difficult to replicate the volatility of house prices in general equilibrium. For example, Morris and Heathcote (2005) generate a volatility that is less than one-third the volatility of house prices in the US data. Our model's ability to generate sufficiently volatile house prices is driven almost exclusively by the presence of financial frictions. In the absence of financial frictions, the model generates a volatility of house prices that is almost ten times too small. More formally, an analysis of the Bayes factors comparing alternative versions of the model supports the inclusion of financial frictions. However, we also find that shocks to housing demand and credit conditions are not particularly important. These results suggest that shocks specifically related to housing are not needed once the borrowing constraint has been accounted for.

The remainder of the paper is organized as follows. Section 2 describes the model. The calibration, data and estimation results are described in sections 3 and 4. In section 5 we offer some concluding remarks.

2 Model

The model can be succinctly characterized as a medium-sized small open economy model with a variety of real, nominal and financial frictions. For the most part, the real and nominal frictions follow Christiano, Eichenbaum and Evans (2005). These include habit persistence in consumption, investment adjustment costs, variable capital utilization and dynamically indexed sticky wages and prices. Financial frictions are introduced along the lines of Iacoviello (2005). A subset of households are assumed to face a binding borrowing constraint that allows them to borrow only up to a fraction of the value of their housing stock. The model includes 11 exogenous shocks: time preference, housing demand, labour supply, credit conditions, productivity, markup, foreign output, foreign nominal interest rate, foreign inflation, risk premium, and monetary policy. The introduction of financial frictions into a model with a rich set of shocks and frictions is a contribution of this paper.

It should be noted that we do not consider all of the shocks to be truly structural. Rather, we prefer to think of some the shocks as wedges as in Chari, Kehoe and McGrattan (2004). That is, we regard some of these shocks are merely measuring the importance of distortions along particular margins for aggregate fluctuations. For example, we model the credit conditions shock as a shift in the proportion of a household's housing stock that can serve as collateral. It is unlikely that this proportion is literally varying exogenously over time. Instead, we would interpret estimation results that assign an important role to this shock as suggesting that some mechanism has been omitted from the model. This would warrant a deeper exploration of the structural determinants of the shifts along this margin. If, on the other hand, the estimation results showed that this shock was not important for aggregate fluctuations, then future research could be focused on distortions along other margins. The same reasoning applies to the housing demand shock.

As previously noted the borrowing constraint introduces a "financial accelerator" effect into the model's dynamics. The financial accelerator effect operates in addition to the traditional wealth effect. Variations in the price of housing affect its collateral value. This in turn affects the borrowing and spending decisions of liquidity-constrained households.

2.1 Household Sector

There are two types of household: patient and impatient. The two types are symmetric except for the fact that impatient households have a higher rate of time preference and face a borrowing constraint. This is a fairly standard analytical device used to generate borrowing and lending within the household sector in representative agent environments.[4] It can be shown that in the vicinity of the non-stochastic steady-state, the desired borrowing of impatient households is unbounded. Following Iacoviello (2005), we assume that impatient households can only borrow up to a fraction of the value of their housing assets. In other words, housing assets are not fully collateralizable. Near the non-stochastic steady-state this constraint is binding.

2.1.1 Patient Households

There is a continuum of patient households. Patient households are fairly standard, with the exception that housing services appear in their utility function. Each household supplies a variety of labour types indexed by $j \in (0,1)$. One can imagine the multiple labour types to be different members of the same household. Patient households' preferences are given by:

$$E_t \sum_{l=0}^{\infty} \beta_p^l \zeta_{\beta,t+l} \left[\log\left(C_{t+l}^p - \gamma C_{t-1+l}^p\right) + \zeta_{H,t+l} \log\left(H_{t+l}^p\right) - \frac{\zeta_{L,t+l}}{2} \int_0^1 \left(L_{j,t+l}^p\right)^2 dj \right] \tag{1}$$

where β_p is the discount factor for patient households, C_t^p is time t consumption, H_t^p is the stock[5] of housing held at the beginning of time t, and $L_{j,t}^p$ denotes time t hours worked of type j. The superscript p indicates that the variable is associated with the patient households. In addition, $\zeta_{\beta,t}$, $\zeta_{H,t}$ and $\zeta_{L,t}$ are, respectively, a time preference shock, a housing demand shock, and a labour supply shock that follow AR(1) processes:

[4] For example, see Iacoviello (2005) and Campbell and Hercowitz (2004).

[5] Our specification of utility assumes that the flow of housing services is proportional to the stock of housing held by a given household.

$$
\begin{aligned}
\log\left(\zeta_{\beta,t+1}\right) &= \left(1-\rho_\beta\right)\log\left(\zeta_\beta\right)+\rho_\beta\log\left(\zeta_{\beta,t}\right)+\varepsilon_{\beta,t+1} && (2)\\
\log\left(\zeta_{H,t+1}\right) &= \left(1-\rho_H\right)\log\left(\zeta_H\right)+\rho_H\log\left(\zeta_{H,t}\right)+\varepsilon_{H,t+1} && (3)\\
\varepsilon_{H,t+1} &= \rho_{\varepsilon_H}\varepsilon_{H,t}+\upsilon_{H,t+1} && (4)\\
\log\left(\zeta_{L,t+1}\right) &= \left(1-\rho_L\right)\log\left(\zeta_L\right)+\rho_L\log\left(\zeta_{L,t}\right)+\varepsilon_{L,t+1} && (5)
\end{aligned}
$$

where $\varepsilon_{\beta,t+1}$, $\upsilon_{H,t+1}$ and $\varepsilon_{L,t+1}$ are i.i.d. normal random variables with mean zero and variances σ_β^2, σ_H^2 and σ_L^2, respectively. The period t budget constraint for a patient household is:

$$
\begin{aligned}
& P_t\left(C_t^p+a\left(u_t\right)\overline{K}_t\right)+R_{t-1}^n B_t^p+\kappa_{t-1}R_{t-1}^{n*}e_t B_t^*+Q_{K,t}\left[\overline{K}_{t+1}-\left(1-\delta_K\right)\overline{K}_t\right]\\
& +Q_{H,t}\left[H_{t+1}^p-\left(1-\delta_H\right)H_t^p+\Phi\left(\frac{H_{t+1}^p}{H_t^p}\right)H_t^p\right] \qquad (6)\\
= & \int_0^1 W_{j,t}^p L_{j,t}^p dj+R_t^k u_t\overline{K}_t+B_{t+1}^p+e_t B_{t+1}^*+\Pi_t
\end{aligned}
$$

where P_t is the price level, u_t is the capital utilization rate, e_t is the nominal exchange rate (the domestic currency price of foreign currency), $\overline{K}_t$ is the capital stock, $Q_{K,t}$ and $Q_{H,t}$ are the nominal prices of capital and housing, δ_K and δ_H are the depreciation rates for capital and housing, B_t^p denotes the nominal quantity of domestic borrowing from period $t-1$ to t, R_{t-1}^n is the nominal interest rate paid on domestic bonds held from $t-1$ to t. Similarly, B_t^* is the nominal quantity of foreign borrowing by domestic patient households, and R_{t-1}^{n*} is the associated foreign nominal interest rate. In addition, $W_{j,t}^p$ is the period t nominal wage of labour type j, R_t^k is the nominal rental rate for capital services, and Π_t is profits from firms. Both the capital utilization cost function, $a(\bullet)$, and the housing adjustment cost function, $\Phi(\bullet)$, are increasing and convex and defined as follows:

$$
a\left(u_t\right)=\gamma_1\left(u_t-1\right)+\frac{\gamma_2}{2}\left(u_t-1\right)^2 \qquad (7)
$$

$$
\Phi\left(\frac{H_{t+1}^p}{H_t^p}\right)=\frac{\phi}{2}\left(\frac{H_{t+1}^p}{H_t^p}-1\right)^2 \qquad (8)
$$

The country-specific risk premium, κ_t, is given by:

$$
\begin{aligned}
\kappa_t &= \exp\left(-\frac{\varphi e_t B_{t+1}^*}{P_t^d Y_t}\right)+\varepsilon_{\kappa,t} && (9)\\
\varepsilon_{\kappa,t+1} &= \rho_\kappa\varepsilon_{\kappa,t}+\upsilon_{\kappa,t+1} && (10)
\end{aligned}
$$

where $\upsilon_{\kappa,t+1}$ is an i.i.d. normal random variable with mean zero and variance σ_κ^2. The risk premium is taken as given by households when they are solving their optimization problems.

In each period the household chooses C_t^p, H_{t+1}^p, $\overline{K}_{t+1}$, u_t, B_{t+1}, B_{t+1}^*. The wage for each type of labour is set by a union that is described below. Labour supply is determined by the requirement that the household meet demand at the prevailing wage. Before discussing wage setting we review the first-order conditions for the other variables.

Following Rotemberg and Woodford (1997), CEE and others we assume that the household chooses its period t consumption prior to observing the period t shocks. Thus, the first-order condition with respect to C_t^p is:

$$\left(\frac{\zeta_{\beta,t}}{C_t^p - \gamma C_{t-1}^p}\right) - \beta_p \gamma E_{t-1}\left(\frac{\zeta_{\beta,t+1}}{C_{t+1}^p - \gamma C_t^p}\right) = E_{t-1}\left[\Lambda_t^p P_t\right] \tag{11}$$

where the Lagrange multiplier on the budget constraint is equal to $\Lambda_t^p/\zeta_{\beta,t}$. It will be useful to define the marginal utility of consumption as:

$$\lambda_t^p \equiv \Lambda_t^p P_t \tag{12}$$

The first-order condition for H_{t+1}^p implies:

$$\begin{aligned} 1+ \quad & \Phi'\left(\frac{H_{t+1}^p}{H_t^p}\right) = \beta_p E_t\left[\frac{\zeta_{\beta,t+1}\zeta_{H,t+1}}{\lambda_t^p q_{H,t} H_{t+1}^p}\right. \\ & \left. + \left(\frac{\lambda_{t+1}^p}{\lambda_t^p}\right)\left(\frac{q_{H,t+1}}{q_{H,t}}\right)\left((1-\delta_H) + \Phi'\left(\frac{H_{t+2}^p}{H_{t+1}^p}\right)\left(\frac{H_{t+2}^p}{H_{t+1}^p}\right) - \Phi\left(\frac{H_{t+2}^p}{H_{t+1}^p}\right)\right)\right] \end{aligned} \tag{13}$$

The first-order condition for B_{t+1}^p implies:

$$1 = \beta_p E_t\left[\left(\frac{\lambda_{t+1}^p}{\lambda_t^p}\right)\frac{R_t^n}{\pi_{t+1}}\right] \tag{14}$$

The first-order condition for $\overline{K}_{t+1}$ implies:

$$1 = \beta_p E_t\left[\left(\frac{\lambda_{t+1}^p}{\lambda_t^p}\right)\left(\frac{r_{t+1}^k u_{t+1} - a\left(u_{t+1}\right) + \left(1-\delta_K\right) q_{K,t+1}}{q_{K,t}}\right)\right] \tag{15}$$

The first-order condition for u_t implies:

$$r_t^k - a'\left(u_t\right) = 0 \tag{16}$$

The first-order condition for B_{t+1}^* implies:

$$1 = \beta_p E_t\left[\left(\frac{\lambda_{t+1}^p}{\lambda_t^p}\right)\kappa_t\left(\frac{R_t^{n*}}{\pi_{t+1}}\right)\left(\frac{e_{t+1}}{e_t}\right)\right] \tag{17}$$

Combining the first-order conditions for B_{t+1} and B_{t+1}^* yields the interest parity condition:

$$E_t\left\{\left(\frac{\lambda_{t+1}^p}{\lambda_t^p}\right)\left[\frac{R_t^n}{\pi_{t+1}} - \kappa_{t+1}\left(\frac{R_t^{n*}}{\pi_{t+1}}\right)\left(\frac{e_{t+1}}{e_t}\right)\right]\right\} = 0 \tag{18}$$

We can now turn to the wage decision. As mentioned above, for each of the j labour types there is a union that is the monopoly supplier of labour type j, $L^p_{j,t}$. It sells this service to a representative, competitive, price-taking firm that transforms it into an aggregate labour input, L^p_t, using the technology:

$$L^p_t = \left[\int_0^1 \left(L^p_{j,t}\right)^{\frac{1}{\theta_w}} dj \right]^{\theta_w} \tag{19}$$

The demand curve for $L^p_{j,t}$ is:

$$L^p_{j,t} = \left(\frac{W^p_{j,t}}{W^p_t} \right)^{\frac{\theta_w}{\theta_w - 1}} L^p_t \tag{20}$$

where W^p_t is the aggregate wage rate, that is, the nominal price of L^p_t. It can be shown that W^p_t is related to $W^p_{j,t}$ by the Dixit-Stiglitz index:

$$W^p_t = \left[\int_0^1 \left(W^p_{j,t}\right)^{\frac{1}{1-\theta_w}} dj \right]^{1-\theta_w} \tag{21}$$

Unions set their nominal wage according to a variant of the Calvo mechanism. Each period unions face a constant probability, $1-\xi_w$, of being able to reoptimize their wage. If they are unable to reoptimize their wage in period t then they set:

$$W^p_{j,t} = \pi^{\chi_w}_{t-1} \overline{\pi}^{1-\chi_w} W^p_{j,t-1} \tag{22}$$

where χ_w measures the degree indexation to past inflation. In particular, if $\chi_w = 1$ the wage is fully indexed to lagged inflation and if $\chi_w = 0$ the wage is indexed to steady-state inflation, $\overline{\pi}$.

Let $\widetilde{W}^p_t$ denote the value of $W^p_{j,t}$ chosen by a union that can reoptimize its wage in period t. The union index, j, is suppressed because all unions that can reoptimize their wage in period t choose the same wage.[6] We assume that the union's objectives coincide with the household's objectives. Thus, the union chooses $\widetilde{W}^p_t$ to maximize:

$$E_t \sum_{l=0}^{\infty} \left(\beta_p \xi_w\right)^l \zeta_{\beta,t+l} \left\{ -\frac{\zeta_{L,t+l}}{2} \left(L^p_{j,t+l}\right)^2 + \frac{\Lambda^p_{t+l}}{\zeta_{\beta,t+l}} W^p_{j,t+l} L^p_{j,t+l} \right\} \tag{23}$$

subject to the labour demand equation, (20). The presence of ξ_w in the discount factor serves to isolate states of the world in which the union has not been able to reoptimize its wage. It is only in these states that the choice of $\widetilde{W}^p_t$ affects utility. Note that if union j resets its wage in period t and has not reoptimized it up to period $t + l$, then, using (22):

$$W^p_{j,t+l} = \widetilde{W}^p_t \overline{\pi}^{(1-\chi_w)l} \prod_{k=1}^{l} \pi^{\chi_w}_{t+k-1} \tag{24}$$

[6] The proof is simply that the first-order condition for $\widetilde{W}^p_t$ does not depend on any variable with a j subscript, that is, it does not depend on any variable that differs across patient unions.

The aggregate nominal wage in period $t+l$ can be written as a function of the real wage $w_{t+l}^{p} = W_{t+l}^{p}/P_{t+l}$, past inflation, and the price level in some base period, t:

$$W_{t+l}^{p} = w_{t+l} P_t \prod_{k=1}^{l} \pi_{t+k} \tag{25}$$

Combining (24) and (25) with (20), we obtain:

$$L_{j,t+l}^{p} = \left(\frac{\widetilde{W}_t^p}{w_{t+l}^{p} P_t} \prod_{k=1}^{l} \left(\frac{\pi_{t+k-1}^{\chi_w} \overline{\pi}^{1-\chi_w}}{\pi_{t+k}} \right) \right)^{\frac{\theta_w}{1-\theta_w}} L_{t+l}^{p} \tag{26}$$

Then we can write the objective as:

$$E_t \sum_{l=0}^{\infty} \left(\beta_p \xi_w\right)^l \left\{ -\frac{\zeta_{\beta,t+l}\zeta_{L,t+l}}{2} \left[\left(\frac{\widetilde{W}_t^p}{w_{t+l}^{p} P_t} X_{t,l}^{w} \right)^{\frac{\theta_w}{1-\theta_w}} L_{t+l}^{p} \right]^2 + \lambda_{t+l}^{p} \frac{\widetilde{W}_t^p}{P_t} X_{t,l}^{w} \left(\frac{\widetilde{W}_t^p}{w_{t+l}^{p} P_t} X_{t,l}^{w} \right)^{\frac{\theta_w}{1-\theta_w}} L_{t+l}^{p} \right\} \tag{27}$$

where

$$X_{t,l}^{w} = \begin{cases} \overline{\pi}^{(1-\chi_w)l} \prod_{k=1}^{l} \left(\frac{\pi_{t+k-1}^{\chi_w}}{\pi_{t+k}} \right) & \text{for } l > 0 \\ 1 & \text{for } l = 0 \end{cases} \tag{28}$$

The first-order condition with respect to $\widetilde{W}_t^p$ implies:

$$E_t \sum_{l=0}^{\infty} \left(\beta_p \xi_w\right)^l \lambda_{t+l}^{p} L_{j,t+l}^{p} \left\{ \widetilde{w}_t^p X_{t,l}^{w} - \theta_w \frac{\zeta_{\beta,t+l}\zeta_{L,t+l} L_{j,t+l}^{p}}{\lambda_{t+l}^{p}} \right\} = 0 \tag{29}$$

where $\widetilde{w}_t^p = \widetilde{W}_t^p / P_t$. Equation (29) states that the union chooses $\widetilde{W}_t^p$ so as to set its expected future real wage as a markup over the expected disutility of labour.

One can then use (21) to show that the aggregate wage is evolves according to:

$$\left(w_t^p\right)^{\frac{1}{1-\theta_w}} = \left(1-\xi_w\right) \left(\widetilde{w}_t^p\right)^{\frac{1}{1-\theta_w}} + \xi_w \left(\frac{\pi_{t-1}^{\chi_w} \overline{\pi}^{1-\chi_w} w_{t-1}^{p}}{\pi_t} \right)^{\frac{1}{1-\theta_w}} \tag{30}$$

2.1.2 Impatient Households

There is a continuum of impatient households. Impatient households discount the future more heavily than patient households, so that $\beta_i < \beta_p$. This difference in discount rates implies that impatient households will want to borrow from patient households in equilibrium. Following Iacoviello (2005), we impose a borrowing constraint on impatient households. Like their patient counterparts, each impatient household supplies a variety of different labour types indexed by $j \in (0,1)$. The preferences of the representative impatient household are given by:

$$E_t \sum_{l=0}^{\infty} \beta_i^l \zeta_{\beta,t+l} \left[\log\left(C_{t+l}^i - \gamma C_{t-1+l}^i\right) + \zeta_{H,t+l} \log\left(H_{t+l}^i\right) - \frac{\zeta_{L,t+l}}{2} \int_0^1 \left(L_{j,t+l}^i\right)^2 dj \right] \tag{31}$$

where β_i is the discount factor for impatient households, C_t^i is time t consumption, H_t^i is the stock of housing held at the beginning of time t, and $L_{j,t}^i$ denotes time t hours worked of type j. The superscript i indicates that the variable is associated with the impatient households. In addition, $\zeta_{\beta,t+l}$, $\zeta_{H,t}$ and $\zeta_{L,t}$ are defined as before. Note that the preference shocks are assumed to be the same for the patient and impatient household.

Near the non-stochastic steady-state impatient households will choose to short sell all assets other than housing. To see this, consider the case of physical capital. From the patient household's Euler equation for capital (eq. (15)), it is apparent that the steady-state return to capital is[7]:

$$r^k = \frac{1}{\beta_p} - (1 - \delta_K) \tag{32}$$

It is impossible for a similar Euler equation to hold for impatient households because $\beta_i < \beta_p$. In particular, in steady-state we would have:

$$r^k < \frac{1}{\beta_i} - (1 - \delta_K) \tag{33}$$

Thus, the impatient households would attempt to short sell capital. Intuitively, in order to hold any given asset, the impatient households will require a higher rate of return than the patient households. We assume that short sales are impossible. This allows us to write the budget constraint for impatient households omitting assets that are not held in equilibrium[8,9]. The period t budget constraint for the impatient household:

$$\begin{aligned} & P_t C_t^i + Q_{H,t} \left[H_{t+1}^i - (1 - \delta_H) H_t^i + \Phi\left(\frac{H_{t+1}^i}{H_t^i}\right) H_t^i \right] + R_{t-1}^n B_t^i \\ = & \int_0^1 W_{j,t}^i L_{j,t}^i dj + B_{t+1}^i \end{aligned} \tag{34}$$

where B_t^i denotes the nominal quantity of borrowing from period $t-1$ to t. Impatient households also face a borrowing constraint of the form:

$$B_{t+1}^i \leq m_t E_t \left[Q_{H,t+1} H_{t+1}^i (1 - \delta_H) \right] \tag{35}$$

$$\log(m_{t+1}) = (1 - \rho_m) \log(m) + \rho_m \log(m_t) + \varepsilon_{m,t+1} \tag{36}$$

$$\varepsilon_{m,t+1} = \rho_{\varepsilon_m} \varepsilon_{m,t} + v_{m,t+1} \tag{37}$$

[7] Here we are using the fact that in steady-state $q_t^K = 1$.

[8] The qualifier "near the non-stochastic steady-state" is meant to be understood.

[9] Profits are omitted from the budget constraint, because impatient households will choose not to hold shares in firms by reasoning analogous to that in the text.

where $m \in (0,1)$ is a parameter and $v_{m,t+1}$ is an i.i.d. normal random variable with mean zero and variance σ_m^2. Our linear solution procedure requires us to assume that the borrowing constraint always binds. Iacoviello (2005) shows that this is, in fact, plausible.[10]

In each period the household chooses C_t^i, H_{t+1}^i, B_{t+1}^i. The mechanism by which the wage is determined is symmetric to that for patient households.

The first-order condition with respect to C_t^i is:

$$\left(\frac{\zeta_{\beta,t}}{C_t^i - \gamma C_{t-1}^i}\right) - \beta_i \gamma E_{t-1}\left(\frac{\zeta_{\beta,t+1}}{C_{t+1}^i - \gamma C_t^i}\right) = E_{t-1}\left[\Lambda_t^i P_t\right] \tag{38}$$

where the Lagrange multiplier on the budget constraint is equal to $\Lambda_t^i/\zeta_{\beta,t}$. It will be useful to define the marginal utility of consumption as:

$$\lambda_t^i \equiv \Lambda_t^i P_t \tag{39}$$

The first-order condition for H_{t+1}^i implies:

$$\begin{aligned} 1 + \Phi'\left(\frac{H_{t+1}^i}{H_t^i}\right) &= \beta_i E_t \Bigg[\frac{\zeta_{\beta,t+1}\zeta_{H,t+1}}{\lambda_t^i q_{H,t} H_{t+1}^i} \\ &+ \left(\frac{\lambda_{t+1}^i}{\lambda_t^i}\right)\left(\frac{q_{H,t+1}}{q_{H,t}}\right)\left((1-\delta_H) + \Phi'\left(\frac{H_{t+2}^i}{H_{t+1}^i}\right)\left(\frac{H_{t+2}^i}{H_{t+1}^i}\right) - \Phi\left(\frac{H_{t+2}^i}{H_{t+1}^i}\right)\right)\Bigg] \\ + \quad & E_t\left[\left(\frac{\mu_t^i}{\lambda_t^i}\right)\left(\frac{q_{H,t+1}}{q_{H,t}}\right)\pi_{t+1} m_t (1-\delta_H)\right] = 0 \end{aligned} \tag{40}$$

where the Lagrange multiplier on the borrowing constraint is equal to $\mu_t^i/\zeta_{\beta,t}$. The first-order condition for B_{t+1}^i implies:

$$1 = \beta_i E_t\left[\left(\frac{\lambda_{t+1}^i}{\lambda_t^i}\right)\frac{R_t^n}{\pi_{t+1}}\right] + \left(\frac{\mu_t^i}{\lambda_t^i}\right) R_t^n \tag{41}$$

Since the wage-setting decision is symmetric to the wage decision of patient households, we do not repeat all the details of the derivation. The first-order condition for each impatient union with respect to $\widetilde{W}_t^i$ implies:

$$E_t \sum_{l=0}^{\infty} (\beta_i \xi_w)^l \lambda_{t+l}^i L_{j,t+l}^i \left\{\widetilde{w}_t^i X_{t,l}^w - \theta_w \frac{\zeta_{\beta,t+l}\zeta_{L,t+l} L_{j,t+l}^i}{\lambda_{t+l}^i}\right\} = 0 \tag{42}$$

where $\widetilde{w}_t^i = \widetilde{W}_t^i/P_t$. Equation (42) states that the union chooses $\widetilde{W}_t^i$ so as to set its expected future real wage as a markup over the expected disutility of labour.

[10] In principle, the borrowing constraint binds in the non-stochastic steady-state and in a neighbourhood of the steady-state. If impatient household wealth were to grow sufficiently large relative to the steady-state, then it would be possible for the borrowing constraint to become non-binding. Iacoviello (2005) shows that this is highly unlikely for reasonable shock variances.

As with the patient unions, one can show that the aggregate wage is evolves according to:

$$\left(w_t^i\right)^{\frac{1}{1-\theta_w}} = (1-\xi_w)\left(\widetilde{w}_t^i\right)^{\frac{1}{1-\theta_w}} + \xi_w \left(\frac{\pi_{t-1}^{\chi_w}\overline{\pi}^{1-\chi_w} w_{t-1}^i}{\pi_t}\right)^{\frac{1}{1-\theta_w}} \tag{43}$$

2.2 Capital Producers

There is a large number of identical price-taking capital producers. Capital producers are owned by patient households and any profits (losses) are transferred in a lump-sum fashion to the patient households. Capital producers purchase investment goods in the final goods market which they transform into capital goods. Capital producers purchase existing capital, $x_{K,t}$, and investment goods, I_t^K, and combine these to produce new capital, $x'_{K,t}$, using the following technology:

$$x'_{K,t} = x_{K,t} + F\left(I_t^K, I_{t-1}^K\right) \tag{44}$$

New capital produced in period t can be used in productive activities in period $t+1$. Let $Q_{K,t}$ be the nominal price of new capital. Since the marginal rate of transformation between new and old capital is unity, the price of old capital is also $Q_{K,t}$. Then the representative capital producer's period t profits are:

$$\Pi_t^K = Q_{K,t}\left[x_{K,t} + F\left(I_t^K, I_{t-1}^K\right)\right] - Q_{K,t}x_{K,t} - P_t I_t^K \tag{45}$$

The profit maximization problem of the capital producer is intertemporal in nature because the period t choice of I_t^K affects profits in period $t+1$. Thus, the profit maximization problem is:

$$\max_{I_t^K} E_{t-1}\left\{\sum_{j=0}^{\infty}\beta_p^j \Lambda_{t+j}^p \left(Q_{K,t+j}\left[x_{K,t+j} + F\left(I_{t+j}^K, I_{t+j-1}^K\right) - Q_{K,t+j}x_{K,t+j} - P_{t+j}I_{t+j}^K\right]\right)\right\} \tag{46}$$

where P_t is the price of final output and Λ_{t+j}^p is the marginal value of a dollar to the patient household in period $t+j$. Note that from this problem it is obvious that any value of $x_{K,t+j}$ is profit maximizing. Thus, the market clearing condition will determine the level of $x_{K,t+j}$:

$$x_{K,t+j} = (1-\delta_K)\overline{K}_{t+j} \tag{47}$$

where $\overline{K}_{t+j}$ is the aggregate capital stock in period $t+j$.

Let the capital investment adjustment cost function be given by:

$$F\left(I_t^K, I_{t-1}^K\right) = \left[1 - S\left(\frac{I_t^K}{I_{t-1}^K}\right)\right] I_t^K \tag{48}$$

$$S\left(\frac{I_t^K}{I_{t-1}^K}\right) = \frac{\psi_K}{2}\left(\frac{I_t^K}{I_{t-1}^K} - 1\right)^2 \tag{49}$$

Period t investment is assumed to be chosen prior to the realization of the period t shocks. Thus, the first-order condition for I_t^K is:

$$E_{t-1}\left[\lambda_t^p q_{K,t} F_1\left(I_t^K, I_{t-1}^K\right) - \lambda_t^p + \beta_p \lambda_{t+1}^p q_{K,t+1} F_2\left(I_{t+1}^K, I_t^K\right)\right] = 0 \tag{50}$$

2.3 Housing Producers

Housing producers are symmetric to capital producers. New housing, $x'_{H,t}$, is produced using the following technology:

$$x'_{H,t} = x_{H,t} + F\left(I_t^H, I_{t-1}^H\right) \tag{51}$$

where the definitions are analogous to the capital production technology. New housing produced in period t yields a flow of housing services in period $t+1$. The representative housing producer's period t profits are:

$$\Pi_t^H = Q_{H,t}\left[x_{H,t} + F\left(I_t^H, I_{t-1}^H\right)\right] - Q_{H,t} x_{H,t} - P_t I_t^H \tag{52}$$

Thus, the profit maximization problem is:

$$\max_{I_t^H} E_{t-1}\left\{\sum_{j=0}^{\infty} \beta_p^j \Lambda_{t+j}^p \left(Q_{H,t+j}\left[x_{H,t+j} + F\left(I_{t+j}^H, I_{t+j-1}^H\right) - Q_{H,t+j} x_{H,t+j} - P_{t+j} I_{t+j}^H\right]\right)\right\} \tag{53}$$

As with the capital producers, the market clearing condition will determine the level of $x_{H,t+j}$:

$$x_{H,t+j} = \left(1 - \delta_H\right) H_{t+j} \tag{54}$$

where H_{t+j} is the aggregate housing stock in period $t+j$.

Let the housing investment adjustment cost function be given by:

$$F\left(I_t^H, I_{t-1}^H\right) = \left[1 - S\left(\frac{I_t^H}{I_{t-1}^H}\right)\right] I_t^H \tag{55}$$

$$S\left(\frac{I_t^H}{I_{t-1}^H}\right) = \frac{\psi_H}{2}\left(\frac{I_t^H}{I_{t-1}^H} - 1\right)^2 \tag{56}$$

Period t investment is assumed to be chosen prior to the realized of the period t shocks. Thus, the first-order condition for I_t^H is:

$$E_{t-1}\left[\lambda_t^p q_{H,t} F_1\left(I_t^H, I_{t-1}^H\right) - \lambda_t^p + \beta_p \lambda_{t+1}^p q_{H,t+1} F_2\left(I_{t+1}^H, I_t^H\right)\right] = 0 \tag{57}$$

2.4 Foreign Economy

We model foreign output, inflation and interest rates (Y_t^*, π_t^*, R_t^{n*}) as block exogenous. These variables are assumed to evolve according to a VAR(2) process. The assumed exogeneity of these variables reflects our belief that the impact of Canadian variables on the rest of the world is sufficiently small that it can be safely ignored. This belief seems reasonable in light of the relatively small size of the Canadian economy.

2.5 Intermediate and Final Goods Firms

The production side of the model is composed of two sectors linked by a vertical input-output structure: a final good sector and an intermediate goods sector. Figure 10 provides a graphical representation. Intermediate goods firms are monopolistically competitive price-setters. They face a constant probability of being able to reoptimize their price in any period. There are two types of intermediate goods firms: domestic and importing firms. The domestic intermediate goods firms use capital and labour as inputs to produce their differentiated goods. The importing firms purchase a homogeneous foreign good on world markets and costlessly differentiate it. The final goods firms use the entire continuum of domestic and foreign intermediate goods to produce a composite final good. The final goods market is perfectly competitive: firms take input prices as given and their output prices are perfectly flexible. The composite final good is used for domestic absorption.

2.5.1 Final Goods Sector

Firms in the final goods sector are perfectly competitive. They combine domestic and imported goods to produce a single homogenous good using the following CES technology:

$$Z_t = \left[(1-\omega)^{\frac{1}{\nu}} \left(Y_t^d \right)^{\frac{\nu-1}{\nu}} + \omega^{\frac{1}{\nu}} \left(Y_t^m \right)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}} \tag{58}$$

where Z_t denotes output of the final good, while $Y_t^d \equiv \left(\int_0^1 Y_t^d(j)^{\frac{\theta_t - 1}{\theta_t}} dj \right)^{\frac{\theta_t}{\theta_t - 1}}$ and $Y_t^m \equiv \left(\int_0^1 Y_t^m(j)^{\frac{\theta_t - 1}{\theta_t}} dj \right)^{\frac{\theta_t}{\theta_t - 1}}$ are composite indexes of domestic and imported intermediate goods, respectively. θ_t is a a mark up shock that follow AR(1) processes:

$$\log(\theta_{t+1}) = (1-\theta) \log(\theta) + \rho_\theta \log(\theta_t) + \varepsilon_{\theta,t+1} \tag{59}$$

Profit maximization, taking the price of the final good P_t as given, implies the set of demand equations for domestic and imported composite goods:

$$Y_t^d = (1-\omega) \left(\frac{P_t^d}{P_t} \right)^{-\nu} Z_t \tag{60}$$

and

$$Y_t^m = \omega \left(\frac{P_t^m}{P_t} \right)^{-\nu} Z_t \tag{61}$$

Thus, as the relative prices of domestic (imported) goods rise, the demand for domestic (imported) goods decreases. It is straightforward to show that the demand for each differentiated domestic intermediate good is given by:

$$Y_t^d(j) = \left(\frac{P_t^d(j)}{P_t^d} \right)^{-\theta_t} Y_t^d \tag{62}$$

Similarly, the demand for each differentiated imported good is:

$$Y_t^m(j) = \left(\frac{P_t^m(j)}{P_t^m} \right)^{-\theta_t} Y_t^m \tag{63}$$

The zero-profit condition implies that the final goods price level is linked to domestic output and import prices through:

$$P_t = \left[(1-\omega) \left(P_t^d \right)^{1-\nu} + \omega \left(P_t^m \right)^{1-\nu} \right]^{1/(1-\nu)}. \tag{64}$$

where $P_t^d \equiv \left(\int_0^1 P_t^d(j)^{1-\theta_t} dj \right)^{\frac{1}{1-\theta_t}}$ and $P_t^m \equiv \left(\int_0^1 P_t^m(j)^{1-\theta_{tt}} dj \right)^{\frac{1}{1-\theta_t}}$ are the price indexes for domestic and imported goods respectively, both expressed in home currency.

2.5.2 Intermediate Goods Sector

Domestic Intermediate Goods Producers The continuum of domestic intermediate goods firms produce differentiated products and behave monopolistically. They have identical Cobb-Douglas production functions given by:

$$Y_t(j) = K_t(j)^{\alpha} \left[A_t N_t(j) \right]^{1-\alpha}, \quad \alpha \in (0,1), \tag{65}$$

where $K_t(j)$ and $N_t(j)$ are capital services and a composite of patient and impatient household labour used by firm j: $N_t(j) \equiv L_t^p(j)^{\eta} L_t^i(j)^{1-\eta}$,with $\eta \in (0,1)$; and A_t is an aggregate technology shock. This shock follows the process

$$\log A_t = (1-\rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{A,t}, \tag{66}$$

where $\rho_A \in (-1,1)$, $A > 0$, and $\varepsilon_{A,t}$ is normally distributed with zero mean and standard deviation σ_A.

As mentioned above, each domestic intermediate good is sold in the domestic market and exported, so that,

$$Y_t(j) = Y_t^d(j) + Y_t^x(j). \tag{67}$$

As mentioned above, the domestic demand for each domestic intermediate good is:

$$Y_t^d(j) = \left(\frac{P_t^d(j)}{P_t^d} \right)^{-\theta_t} Y_t^d \tag{68}$$

We assume that the foreign demand for each domestic intermediate good takes the same form as the domestic demand:

$$Y_t^x(j) = \left(\frac{P_t^d(j)}{P_t^d} \right)^{-\theta_t} Y_t^x \tag{69}$$

where $Y_t^x = \left(\frac{P_t^d}{e_t P_t^*}\right)^{-\tau} Y_t^*$ is aggregate exports[11], and Y_t^* is overall world output.

Domestic intermediate goods producers are monopolistically competitive and face a demand functions for goods given by (68) and (69). As in Calvo (1983), firms cannot change their prices unless they receive a random signal. The probability that a given price can be reset in any period is constant and is given by $(1-\xi_p)$. This implies that on average the price is not re-optimized for $1/(1-\xi_p)$ periods. Further, following CEE and Smets and Wouters (2004), we assume that prices of firms that do not receive a price signal are partially indexed to last period's inflation rate. With probability $\xi_p$the firm is not allowed to re-optimize, and its price in period t is then updated according to the scheme $P_t^d(j) = \pi_{t-1}^{\chi_p}\overline{\pi}^{1-\chi_p} P_{t-1}^d(j)$. Thus, in periods, when domestic producer j is allowed to change its price, it chooses $K_t(j)$, $L_t^p(j)$, and $L_t^i(j)$, and sets the price $\widetilde{P}_t^d$ that maximize the expected discounted flow of its profits[12]. It chooses $\widetilde{P}_t^d$ prior to the realization of the period t shocks. After observing the period t shocks, it chooses $K_t(j)$, $L_t^p(j)$, and $L_t^i(j)$. The firm's objective function is:

$$E_t\left[\sum_{l=0}^{\infty}(\beta_p\xi_p)^l \lambda_{t+l}^p D_{t+l}^d(j)/P_{t+l}\right], \tag{70}$$

subject to (65), (68) and (69). Period profits are given by:

$$D_{t+l}^d(j) = \widetilde{P}_t^d(j)Y_{t+l}(j) - R_{t+l}^k K_{t+l}(j) - W_{t+l}^p L_{t+l}^p(j) - W_{t+l}^i L_{t+l}^i(j) \tag{71}$$

The patient households own all firms so the domestic producer's discount factor is given by the stochastic process $(\beta^l \lambda_{t+l}^p)$, where λ_{t+l}^p denotes the marginal utility of consumption to the patient household in period $t+l$.

The first-order conditions are:

$$\frac{R_t^k}{P_t} = \alpha \frac{Y_t(j)}{K_t(j)} mc_t \tag{72}$$

$$\frac{W_t^p}{P_t} = (1-\alpha)\eta \frac{Y_t(j)}{L_t^p(j)} mc_t \tag{73}$$

$$\frac{W_t^i}{P_t} = (1-\alpha)(1-\eta)\frac{Y_t(j)}{L_t^i(j)} mc_t \tag{74}$$

$$E_{t-1}\sum_{l=0}^{\infty}(\beta_p\xi_p)^l \lambda_{t+l}^p \left(\frac{\widetilde{P}_t^d}{P_t^d}\right)^{-\theta_t} \left(X_{t,l}^p\right)^{-\theta_t} Y_{t+l}\left[\left(\frac{\widetilde{P}_t^d}{P_t^d}\right) X_{t,l}^p - \left(\frac{\theta_t}{\theta_t - 1}\right) mc_{t+l}\right] = 0 \tag{75}$$

[11] Here we assume that exports are invoiced in currency of importing country, which implies gradual pass-through of foreigns to domestic prices. Devreux and Engle (2001), among others, have emphasized the role of local currency pricing.

[12] As in the case of sticky wages discussed above, our notation reflects the fact that all firms that can reoptimize in period t choose the same price.

where mc_t is the Lagrange multiplier associated with the production function constraint and

$$X_{t,l}^p = \begin{cases} \overline{\pi}^{(1-\chi_p)l} \prod_{k=1}^{l} \left(\frac{\pi_{t+k-1}^{\chi_p}}{\pi_{t+k}} \right) & \text{for } l > 0 \\ 1 & \text{for } l = 0 \end{cases} \tag{76}$$

The aggregate domestic price is evolves according to:

$$\left(p_t^d\right)^{1-\theta_t} = \xi_p \left(\frac{\pi_{t-1}^{\chi_p} \overline{\pi}^{1-\chi_p} p_{t-1}^d}{\pi_t} \right)^{1-\theta_t} + (1-\xi_p) \left(\widetilde{p}_t^d\right)^{1-\theta_t}. \tag{77}$$

where $p_t^d = P_t^d / P_t$ and $\widetilde{p_t^d} = \widetilde{P_t^d} / P_t$.

The equations (72), (73), and (74) state that the marginal cost of the inputs should equal to their marginal product weighted by the real marginal cost. The equation (75) relates the optimal price to the expected future price of the final good and to expected future real marginal costs. This condition together with (77) allows us to derive a new Phillips curve.

Imported Intermediate Goods Producers The import sector consists of a continuum of importing firms that buy a homogenous good in the world market for the foreign price P_t^*. Each importer turns this imported product into a differentiated good, $Y_t^m(j)$, which is sold in a domestic monopolistically competitive market to produce the imported composite good Y_t^m. As in the domestic intermediate-goods sector, importing firms can only change their prices when they receive a random signal. The constant probability of receiving such a signal is $(1-\xi_p)$.

When an importer j is allowed to change its price, it sets the price $\widetilde{P_t^m}(j)$ that maximizes its weighted expected profits, given the price of the imported-composite output, P_t^m, the nominal exchange rate, e_t, and the foreign price level, P_t^*. The optimization problem is:

$$\max_{\{\widetilde{P_t^m}(j)\}} E_{t-1} \left[\sum_{t=0}^{\infty} (\beta_p \xi_p)^l \lambda_{t+l}^p D_{t+l}^m(j) / P_{t+l} \right], \tag{78}$$

subject to

$$Y_{t+l}^m(j) = \left(\frac{\widetilde{P_t^m}(j)}{P_{t+l}^m} \right)^{-\theta_t} Y_{t+l}^m, \tag{79}$$

where period profit is given by:

$$D_{t+l}^m(j) = \left(\widetilde{P_t^m}(j) - e_{t+l} P_{t+l}^* \right) Y_{t+l}^m(j). \tag{80}$$

In period t, the importer's nominal marginal cost is $e_t P_t^*$, so that its real marginal cost is the real exchange rate $s_t = e_t P_t^* / P_t$. The importer's discount factor is also given by the stochastic process $(\beta_p^l \lambda_{t+l}^p)$. The first-order condition of this optimization problem is:

$$E_t \sum_{l=0}^{\infty} (\beta_p \xi_p)^l \lambda_{t+l}^p \left(\frac{\widetilde{P}_t^m}{P_t^m}\right)^{-\theta_t} \left(X_{t,l}^p\right)^{-\theta_t} Y_{t+l}^m \left[\left(\frac{\widetilde{P}_t^m}{P_t^m}\right) X_{t,l}^p - \left(\frac{\theta_t}{\theta_t - 1}\right) s_{t+l}\right] = 0 \quad (81)$$

The aggregate import price evolves according to:

$$(p_t^m)^{1-\theta_t} = \xi_p \left(\frac{\pi_{t-1}^{\chi_p} \overline{\pi}^{1-\chi_p} p_{t-1}^m}{\pi_t}\right)^{1-\theta_t} + (1-\xi_p)\left(\widetilde{p_t^m}\right)^{1-\theta_t} \quad (82)$$

2.6 Monetary Authority

The monetary authority is assumed to set nominal interest rates according to a simple reaction function:

$$R_t^n = (1-\rho_R)\,\overline{R^n} + \rho_R R_{t-1}^n + (1-\rho_R)\, R_t^{n,\text{target}} + \varepsilon_{R,t} \quad (83)$$

$$R_t^{n,\text{target}} = (1+\gamma_\pi)\, E_t\left(\widetilde{\pi}_{t+1} - \overline{\pi}\right) + \gamma_y \left(\frac{Y_t}{Y_{t-1}} - 1\right) + \gamma_{q_H}\left(q_{H,t} - 1\right) \quad (84)$$

where $\rho_R \in (0,1)$ is the partial adjustment parameter, $\gamma_\pi > 0$ governs the policy response to expected inflation deviations from target, γ_y governs the response to output growth and γ_{q_H} governs the response to real house price growth. The monetary policy shock, $\varepsilon_{R,t}$, follows an AR(1) process:

$$\varepsilon_{R,t+1} = \rho_{\varepsilon_R} \varepsilon_{R,t} + v_{R,t+1} \quad (85)$$

where $v_{R,t+1}$ is a normal i.i.d. random variable with mean zero and variance σ_R^2.

The targeted inflation measure is:

$$\widetilde{\pi}_t \equiv \frac{(1-\omega_H)\, P_t + \omega_H Q_{H,t}}{(1-\omega_H)\, P_{t-1} + \omega_H Q_{H,t-1}} \quad (86)$$

This is the inflation rate for a price index that includes house prices with a weight ω_H. The inflation rate targeted by the Bank of Canada includes house prices in a similar manner. Our specification of $\widetilde{\pi}_t$ is meant to mimic the inflation measure targeted by the Bank.

3 Calibration

Several of the model's parameter values are fixed prior to estimation. The degree of habit formation, γ, is set to 0.75. This is close to the point estimate of 0.70

reported in Boldrin, Christiano, and Fisher (2001) and it is near the midpoint of the values reported by CEE (0.63), and Fuhrer (2000) (0.8 to 0.9) for the U.S. economy. The subjective discount factor for the patient household, β_p, equals π/R^n. The steady-state inflation rate is set at 2 per cent per annum to match the current inflation target in Canada.[13] β_p is chosen so that the model's steady-state real interest rate is 2.5 per cent per annum. The subjective discount factor for the impatient household, β_i, is calibrated to 0.985. The weight on housing services in the utility function, ζ_H, is set to 0.1, which yields a consumption-to-housing ratio of about 40 per cent (in line with the Canadian data). The housing depreciation rate, δ_H, is set to 0.005 consistent with Statistics Canada's estimate of 2 per cent annual depreciation of housing. The depreciation rate of physical capital, δ_k, and the elasticity of output with respect to capital in the intermediate-good sector, α, are fixed to 0.025 and 0.36, respectively. Canadian micro data indicate that the standard loan-to-value ratio, m, is 0.75. The labour share of patient households, η, is assumed to be 0.5. This is roughly consistent with the fact that about half of Canadian households have mortgage debt. Following the estimates in CEE (2005), we set the elasticity of substitution across different individual labour types, θ_w, to be 1.05. The share of of the domestic composite good in the final good aggregator, ω, is chosen to match the average ratio of Canadian import-to-GDP of 28 per cent. The elasticity of substitution between domestic and imported intermediate goods, ν, and the parameter, τ, determining price-elasticity of world demand for domestic output, are calibrated to 0.8, as estimated by Dib (2003). The elasticity of substitution between domestic (imported) intermediate goods, θ, is calibrated to 6, which implies a steady-state markup of price over marginal cost equal to 20 per cent. This value is roughly consistent with estimates in the empirical literature. Basu (1995) report estimates ranging from 10 per cent to 20 per cent. The parameter γ_2 is chosen to generate an elasticity of capacity utilization with respect to the rental rate of capital of approximately 20 as in Adolfson et al. (2005). The parameter in the risk-premium term, φ, is set equal to 0.0054 implying an average risk premium, κ, of 98 basis points at an annual rate as in Clinton (1999). Finally, the weight of house prices in the targeted inflation rate, ω_H, is set to 0.0329 as in the Canadian Consumer Price Index. Calibrated parameters are summarized in Table 1.

4 Estimation

We employ a Bayesian estimation technique to estimate a subset of the model's parameters.[14] We use ten series in the estimation: Canadian output, consumption, residential investment, hours worked, inflation, nominal interest rates and

[13] In our empirical work we use the target-adjusted inflation series from Amano and Murchison (2005) in order to be consistent with this assumption about steady-state inflation.

[14] Note that the model is first linearized around its non-stochastic steady-state, then standard methods for solving and estimating linear models are used. Much of the work reported in this paper was conducted using the Dynare implementations of these standard algorithms by Michel Juillard.

house prices, as well as US output, inflation and nominal interest rates. The Canadian inflation series is core Consumer Price Index inflation. This measure of inflation plays a key role in policy decisions at the Bank of Canada. In contrast to the US, Canadian core CPI includes a measure of house prices. House prices enter the index with a weight of 0.0329. It should be noted that the inflation and nominal interest rate data we use have been adjusted for variation in the historical inflation target as reported in Amano and Murchison (2005).[15] As Amano and Murchison note, this adjustment substantially reduces the degree of persistence in these series. For Canadian house prices we have a choice between two series: a price index for new houses and an index for existing houses. Of course, in the model, new and existing house prices are equal. Thus, the model offers no guidance for discriminating between measures. The two series are, however, highly correlated.[16] Also, militating in favour of the new house price index is the fact that this is the measure of house prices used in the CPI. Using new house prices to measure house prices would facilitate construction of a comparable CPI measure in the model. Finally, the new house price series is the only series available at a quarterly frequency prior to 1986. Given these considerations we choose the Statistics Canada New House Price Index. The remaining data series are described in more detail in Appendix A. The sample period is 1981Q1-2004Q1.

In order to empirically assess the role of the financial frictions, we estimate three versions of the model: (1) the baseline model as described above (Model 1), (2) a version that eliminates credit shocks ($\sigma_m = 0$) but retains the financial frictions (Model 2), and, (3) a version that eliminates impatient households altogether (Model 3). Since it is the impatient households who face the borrowing constraint, Model 3 naturally does not include any financial frictions. Note that the financial frictions could not be eliminated by simply dropping the borrowing constraint. Some mechanism is needed to keep the borrowing of the impatient households bounded. If impatient households did not face any type of financing friction they would accumulate debt without bound.[17]

The prior and posterior distributions for the three models are summarized in Tables 5 to 12. The same prior distributions are used for the all three models. The priors for the parameters of the policy rule (ρ_R, γ_π, γ_y, γ_{q_H}) reflect several characteristics of policy over the sample period that we believe are fairly widely accepted: (1) moderate interest rate smoothing, (2) a fairly aggressive response to expected inflation, (3) a mild response to output, and (4) no response to house prices. The priors for the adjustment cost parameters (ψ_K, ψ_H) follow Smets and Wouters (2004). The Calvo price and wage stickiness parameters (ξ_p, ξ_w) are chosen to be consistent with prior evidence that prices

[15] Amano and Murchison (2005) construct the historical series for the inflation target by (1) using the announced target in the post-1991 period, and (2) examining internal Bank of Canada projections for the 1981-1991 period.

[16] That is, after removing linear trends from the logarithms of the two series their correlation coefficient is 0.92. The differential trends are due to the fact that the new house price series is quality-adjusted, while the existing house price series is not quality adjusted.

[17] In a similar model, Basant Roi and Mendes (2005) use a debt-elastic risk premium to keep the borrowing of the impatient households bounded.

are reoptimized every 2 quarters on average and wages are reoptimized every 4 quarters on average. The priors for the degrees of indexation (χ_p, χ_w) capture our prior belief that indexation to lagged inflation is more important for wages than prices. The prior for the capital utilization parameter, γ_2, is chosen to yield an elasticity of the utilization rate with respect to the rental rate of capital of about 100 following CEE. The priors for the persistence parameters of the exogenous shocks (excluding the policy shock) are set to 0.75. This is lower than the value of 0.85 used by Smets and Wouters (2004) and others. We justify this lower value by appealing to the finding of Adolfson et al. (2005) that the persistence of the exogenous shocks tends to be lower when open economy features are included in the model. The prior for the persistence of the policy shock, ρ_{ε_R}, is set to 0.2 to reflect our belief that policy shocks tend to have very little persistence.

We can use Bayes factors to compare the fit of our three models. The Bayes factor comparing two models is simply the ratio of the marginal likelihoods of the two models. The marginal likelihood of model i is:

$$M_i = \int L_i\left(\theta_i; x\right) p_i\left(\theta_i\right) d\theta_i \tag{87}$$

where $L_i\left(\theta_i; x\right)$ is likelihood function of the model's parameter vector, θ_i, conditional on the observed data x and $p_i(\theta_i)$ is the prior distribution. The Bayes factor for comparing two competing models is defined as $B(M_i, M_j) = M_i/M_j$. We compute the marginal likelihood numerically from the posterior distribution using the modified harmonic estimator of Geweke (1999). The estimation results support inclusion of financial frictions in the model under the assumed priors.

The Bayes factor for Model 1 versus Model 3 is $B(M_1, M_3) = 162750$ in favour of Model 1. According to the Jeffreys scale[18] anything over 150 provides "very strong" evidence. Thus, we can conclude that the model with financial frictions and credit shocks (Model 1) fits the data substantially better than the model without either of these features (Model 3). Next we want to ask whether it is the financial frictions or the credit shocks or both that allow Model 1 to fit better than Model 3. To do this we compute the Bayes factor for Model 2 versus Model 1 (recall that Model 2 includes the borrowing constraint, but excludes credit shocks). The Bayes factor is $B(M_2, M_1) = 4404$ in favour of Model 2. Thus, the model with no credit shocks fits better than the model with both financial frictions and credit shocks. We conclude that it is the addition of financial frictions that produce the better fit, not the addition of credit shocks.

Tables 13 to 15 show the variance decompositions for the three models. Most striking is the change in the role of monetary policy shocks in generating house price variability. In Models 1 and 2, respectively, monetary policy shocks account for 79 and 65 per cent of the variability of house prices. When financial frictions are eliminated (Model 3) that number drops to less than 12 per cent. Instead, Model 3 attributes more than half the variation in house prices to

[18] Jeffreys (1961, Appendix B)

housing demand shocks and labour supply shocks. The single most important shock for house prices in Model 3 is the housing demand shock at 34 per cent. We interpret these housing demand shocks as wedges in the spirit of Chari, Kehoe and McGrattan (2004). It seems unlikely that preferences for housing services would be very volatile. Instead, the large estimated variance of the housing demand shocks in Model 3 is indicative of some unmodelled feature of the housing market. By comparing Model 3 with Models 1 and 2, we have shown that financial frictions are a good candidate for this unmodelled feature.

The large role of housing demand shocks in Model 3 reflects differences in the transmission mechanism across models. In Models 1 and 2 there is an active credit channel: the reaction of borrowing constrained households amplifies the effects of monetary policy shocks on house prices. For example, in response to a contractionary monetary policy shock, house prices will fall, tightening the borrowing constraint faced by impatient households. Given the tighter borrowing constraint, impatient households desire a lower stock of housing, further depressing house prices[19]. It is through this feedback mechanism that the credit channel amplifies the effects of monetary policy shocks. The absence of a credit channel from Model 3 means that the transmission of monetary policy shocks to house prices is much weaker. Instead, Model 3 must rely on direct shocks to housing demand to generate house price volatility. However, as discussed below, even with substantial housing demand shocks, Model 3 is not able to replicate the observed unconditional variance of house prices.

In contrast with our priors and the previous literature, our estimates of ξ_p and ξ_w in Models 1 and 2 suggest that prices are reoptimized less often than wages. This may be due to the fact that we map the inflation data to a model counterpart that includes house prices:

$$\widetilde{\pi}_t \equiv \frac{(1-\omega_H)\,P_t + \omega_H Q_{H,t}}{(1-\omega_H)\,P_{t-1} + \omega_H Q_{H,t-1}}$$

House prices are excessively flexible in the model. By raising the value of ξ_p, the estimation procedure makes other prices more sticky in order to compensate for the flexibility of house prices.

The policy parameter governing the response to house prices, γ_{q_H}, is positive and significant in all three models. This may appear to suggest that the Bank of Canada was responding directly to house prices, over and above its response to an inflation measure that includes house prices. This, however, is in conflict with much anecdotal evidence. The most likely explanation for the positive estimates of γ_{q_H} is that the two large disinflations in our sample coincided with periods of elevated house prices.[20] The disinflations occurred in the early

[19] This need not have any effect on residential investment as long as the patient households are willing to purchase the additional supply of housing units from the impatient households. In fact, this is borne out by the small contribution of monetary policy shocks to residential investment variability.

[20] There are other possible explanations. For example, we assume that the Bank responds to deviations of output growth from trend, but in practice it appears that the Bank responds to a different measure of the output gap. It is also possible that the Bank has historically

1980s and the early 1990s. At both times real house prices were well above their average for the sample period. Presumably, interest rates responded more strongly to inflation during the disinflations. In our model we assume that the degree of anti-inflation aggresiveness is a constant, γ_π. If, in fact, γ_π was larger than normal during the disinflations, then our constant-coefficient estimation procedure will attribute some of the policy tightness to a reaction to above-average house prices.

The estimated investment adjustment costs are much higher in the absence of financial frictions. In particular, ψ_H is equal to 3.88 in Model 3, while it is 0.52 and 0.13 in Models 1 and 2, respectively. To gain some intuition, note that log-linearizing the first-order condition for the housing investment firms, (57), yields:

$$E_{t-1}\widehat{q}_{H,t} = \psi_H \left[\left(\widehat{I}_t^H - \widehat{I}_{t-1}^H \right) + \beta_p E_{t-1} \left(\widehat{I}_{t+1}^H - \widehat{I}_t^H \right) \right] \tag{88}$$

rearranging,

$$\widehat{I}_t^H = \widehat{I}_{t-1}^H + \frac{1}{\psi_H} \sum_{j=0}^{\infty} \beta_p^j E_{t-1} \widehat{q}_{H,t+j} \tag{89}$$

Thus, $1/\psi_H$ is the elasticity of residential investment with respect to a temporary (one period) increase in the price of housing. A smaller value of ψ_H means that investment is less sensitive to changes in house prices. The estimates of ψ_H are lower for the models with financial frictions because these models endogenously generate much more house price volatility. If ψ_H were larger, then these models would imply too much investment volatility. An alternative way of looking at this is that the models with financial frictions do not require very large adjustment costs to generate house price volatility.

We now consider some of the unconditional properties of the models. The model second moments in the tables are the theoretical moments from the linearized model given the estimated variances. The steady-state properties of the models are very similar and don't require any further comment (see Table 2).

Table 3 displays the standard deviations of some key variables relative to the standard deviation of output. Model 2 fairs better than the other two models in matching the variability of consumption and residential investment, but it overpredicts the volatility of business investment. Model 3, without financially constrained households, fails to replicate the variability of house prices. The relative variability of house prices in Model 3 is almost an order of magnitude less than in the data and less than one quarter of the variability in the models with financial frictions. As discussed above, this is due to the lack of a credit channel in Model 3. Interestingly, Davis and Heathcote (2005), who study a real business cycle model that includes housing, also fail to match the relative variability of house prices in the United States. Their model matches several second moment properties related to housing and the macroeconomy quite well.

responded to credit growth, not house prices. Since we omit credit growth from the reaction function, we may incorrectly attribute some policy actions to house prices.

However, it generates a a relative variability of house prices of only 0.40. The comparable number in the US data is 1.37.[21] Davis and Heathcote's model does not include any financial frictions. This reinforces the point that our model's ability to match the variability of house prices is being driven by the inclusion of financial frictions.

According to Table 4, Models 1 and 2 do a better job of matching correlations of consumption and output. This is not surprising given that these models include financially constrained households who cannot use international capital markets to protect against variation in domestic income. Models 1 and 2 also generate more realistic, though somewhat overstated, correlations between consumption and house prices than Model 3. This has an obvious interpretation. Movements in house prices change the amount that the financially constrained households in Models 1 and 2 can borrow. When house prices rise, these households tend to borrow and consume more. Model 3 does noticeably outperform the financial frictions models on one dimension: the correlations of consumption with residential and business investment. This is due to the fact that time preference shocks play a much more important role in driving consumption and residential investment in Models 1 and 2 than in Model 3 (see Tables 13 to 15). This type of shock drives consumption and residential investment in opposite directions (see Figure 5).

Figures 1 to 9 display some selected impulse responses. Unless otherwise noted, these are the responses of Model 1. The response of output and its major components to a productivity shock is fairly standard as can be seen from Figure 1. Inflation exhibits a small but sharp decline before rising slightly above trend. As households feel wealthier they increase their demand for housing, driving up the price of housing by somewhat less than one per cent. The demand for housing is also reflected in the hump-shaped increase in residential investment. The stock of mortgage credit rises very persistently, reflecting a larger stock of housing.

According to Figure 2, a monetary policy shock lowers output and inflation. The real exchange rate appreciates as expected. This leads to a rapid movement in net exports, explaining the sharp response of output. House prices are very sensitive to monetary policy shocks. A one standard deviation shock – which is equal to about 250 basis points on an annualized basis – causes a decline in real house prices of almost 10 percent. Mortgage credit falls more than two per cent and remains persistently below its steady-state level.

Figure 3 reports the economy's response to credit shocks. An easing of credit constraints allows impatient households to borrow more given their housing stock. This leads directly to an increase in mortgage credit. Impatient households use these additional funds to increase their consumption and residential investment demand. The sharp rise in output is in contrast to the hump-shaped increases in consumption and residential investment. This is again due to a sharp increase in net exports. The increase in domestic credit causes a capital account surplus and exchange rate depreciation. The depreciation, in turn,

[21] This is for the period 1970 to 2001. See Davis and Heathcote (2005) for details.

causes the sharp rise in net exports.

The impulse responses to housing demand shocks are depicted in Figure 4. The housing demand shocks do not play an important role in the Models 1 and 2, but they do play an important role in Model 3. Hence, we show the response of Model 3 to a housing demand shock. As one might expect, real house prices and residential investment rise. Consumption falls as households shift expenditures from consumption to housing. Households also repatriate funds that had been invested abroad in order to invest in housing. This causes a sharp real appreciation of the exchange rate and a decline in net exports, hence the sharp decline in output.

A time preference shock is followed by a persistent rise in current consumption as can be seen in Figure 5. Investment in housing and residential investment falls, which is reflected in the declining level of output. Inflation rises because the preference shock causes households to reduce their current labour supply (increase their consumption of current leisure) driving up wages and marginal cost. House prices fall because households now place less value of the flow of future utility from housing services.

Figure 6 shows that the markup shock causes inflation and output to move in opposite directions as expected. Residential investment rises after a markup shock. House prices decline initially before rising slightly above trend. Thus, they substitute away from housing and toward current consumption and leisure.

From Figure 7 we can see that the labour supply shock has the expected impact on inflation, output and its major components. House prices fall slightly as households substitute away from housing toward leisure.

Figure 8 reports the impulse responses to the country-specific risk premium shock. A risk premium shock causes a depreciation of the real exchange rate, thereby stimulating export demand and causing a sharp rise in output. Inflation rises as would be expected in response to the higher level of aggregate demand. Interestingly, house prices fall in response to the higher risk premium. This is not surprising. The higher risk premium raises the domestic nominal interest rate. The higher interest rate leads households to discount future returns on housing (the flow of utility) more rapidly. Households reduce their desired stock of housing in order to raise the expected future marginal utility of housing.

Finally, Figure 9 plots the dynamic responses to a foreign output shock. A foreign output shock operates principally through export demand (but also indirectly through the interest parity condition). Inflation, output and its components move as expected. Interestingly, house prices fall temporarily. This is due to the higher interest rates that go along with a shock to foreign output.

5 Conclusion

We have presented a model of the Canadian economy that incorporates financial frictions in the household sector. An analysis of Bayes factors strongly supports the inclusion of financial frictions in the household sector. This result is driven by the fact that the model with financial frictions is able to fit the data better on several important dimensions. In particular, the financial frictions generate significant house price volatility and a positive correlation between consumption and house prices. In the absence of financial frictions, our estimation procedure is forced to make the housing demand shock much more variable. This reflects the loss of the internal propagation mechanism provided by the borrowing constraint.

There are, however, several remaining issues. Most prominently, the forward-looking determination of house prices in the model implies that house prices respond very rapidly to shocks. The model impulse responses suggest that house prices exhibit strong mean reversion. In the data, house prices are very persistent. These aspects of the behaviour of house prices in the model are not related to the presence or absence of financial frictions. Rather, they are due to the fact that we treat housing as a fairly standard asset. The main culprits are the twin assumptions of continuity and homogeneity: we treat each household's housing stock as a continuous variable, and we assume that housing units are homogenous throughout the economy. Eliminating these assumptions while preserving the tractability of a model of this size is no mean task. Nevertheless, we believe that our results provide a strong impetus for future work investigating the role of household sector financial frictions. This future work should focus on improving the microeconomic foundations of household sector financial frictions and on eliminating the twin assumptions mentioned above.

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Table 1: Calibrated Parameter Values

Parameter Description	Parameter	Value
Degree of habit formation	γ	0.75
Subjective discount factor for the patient household	β_p	0.994
Subjective discount factor for the impatient household	β_i	0.985
Weight on housing services in utility function	ζ_H	0.1
Housing depreciation rate	δ_H	0.005
Capital depreciation rate	δ_k	0.025
Share of capital in production function	α	0.36
Loan-to-value ratio	m	0.75
Parameter governing housing adjustment cost	ϕ	10
Parameter governing capacity adjustment costs	γ_1	0.0310
Fraction of patient household	η	0.5
Elasticity of substitution across different labour types	θ_w	1.05
Share of of the domestic composite good in the final good aggregator	ω	0.28
Elasticity of substitution between domestic and imported goods	ν	0.8
Elasticity of substitution between domestic intermediate goods	θ	6
Price-elasticity of world demand for domestic output	τ	0.8
Weight of house prices in targeted inflation measure	ω_H	0.0329
Parameter in the risk-premium term	φ	0.0054

Table 2: Properties of Steady-state

	Data (1981-2004)	Model 1	Model 2	Model 3
Consumption-to-GDP	0.573	0.695	0.695	0.694
Housing Stock-to-GDP	1.287	1.426	1.426	1.542
Residential Investment-to-GDP	0.055	0.030	0.030	0.031
Mortgage Credit-to-Housing Stock	0.319	0.263	0.263	N/A

Table 3: Relative Standard Deviations (relative to SD of Output)

	Data (1981-2004)	Model 1	Model 2	Model 3
Real House Prices	1.84	1.45	1.40	0.29
Consumption	0.84	0.60	0.91	0.67
Residential Investment	4.45	2.86	4.36	1.81
Business Investment	3.26	4.40	7.97	2.81

Table 4: Model and data correlations

	Data (1981-2004)	Model 1	Model 2	Model 3
$corr(Y_t,C_t)$	0.81	0.63	0.70	0.37
$corr(q_{H,t},I_t^H)$	0.13	0.19	0.03	0.22
$corr(q_{H,t},C_t)$	0.43	0.69	0.52	0.17
$corr(C_t,I_t^H)$	0.807	0.30	0.16	0.69
$corr(C_t,I_t^K)$	0.689	0.03	0.13	0.81
$corr(I_t^H,I_t^K)$	0.298	0.64	0.64	0.74

Table 5: Priors

Parameter	Prior Mean	Prior S.D.	Prior Distribution
ρ_R	0.65	0.1	beta
γ_π	0.7	0.1	gamma
γ_y	0.25	0.1	gamma
γ_{q_H}	0	0.2	normal
χ_w	0.75	0.15	beta
χ_p	0.25	0.15	beta
ξ_p	0.5	0.1	beta
ξ_w	0.75	0.1	beta
ψ_K	4	1.5	normal
ψ_H	4	1.5	normal
ρ_A	0.75	0.1	beta
ρ_κ	0.75	0.1	beta
ρ_{ε_H}	0.75	0.1	beta
ρ_H	0.75	0.1	beta
ρ_L	0.75	0.1	beta
ρ_m	0.75	0.1	beta
ρ_{ε_m}	0.75	0.1	beta
ρ_β	0.75	0.1	beta
ρ_θ	0.75	0.1	beta
ρ_{ε_R}	0.2	0.1	beta

Table 6: Priors for standard Deviations

Parameter	Prior Mean	Prior S.D.	Prior Distribution
σ_A	0.02	2	inverse gamma
σ_κ	0.0025	2	inverse gamma
σ_R	0.02	2	inverse gamma
σ_H	0.02	2	inverse gamma
σ_L	0.02	2	inverse gamma
σ_m	0.02	2	inverse gamma
σ_β	0.0025	2	inverse gamma
σ_θ	0.02	2	inverse gamma

Table 7: Posterior Distribution Model 1 (Financial Frictions and Credit Shocks)

Parameter	Posterior Mean	5th Percentile	95th Percentile
ρ_R	0.7146	0.6156	0.7707
γ_π	0.8502	0.7962	0.9846
γ_y	0.2735	0.2184	0.3509
γ_{q_H}	0.2509	0.2319	0.2627
χ_w	0.4520	0.2923	0.8010
χ_p	0.1406	0.0458	0.1938
ξ_p	0.8264	0.8101	0.8430
ξ_w	0.6744	0.6135	0.7908
ψ_K	1.1499	0.1207	3.4091
ψ_H	0.5215	0.2704	0.7479
ρ_A	0.8578	0.8493	0.9099
ρ_κ	0.9255	0.8797	0.9440
ρ_{ε_H}	0.7782	0.6994	0.8541
ρ_H	0.7714	0.7614	0.7708
ρ_L	0.7746	0.6955	0.8430
ρ_m	0.6645	0.6577	0.6982
ρ_{ε_m}	0.6147	0.5737	0.6592
ρ_β	0.9761	0.9717	0.9772
ρ_θ	0.7697	0.6974	0.8327
ρ_{ε_R}	0.0664	0.0333	0.1429

Table 8: Posterior Distributions for S.D. (Model 1)

Parameter	Posterior Mean	5th Percentile	95th Percentile
σ_A	0.0104	0.0094	0.0108
σ_κ	0.0026	0.0022	0.0030
σ_R	0.0072	0.0052	0.0086
σ_H	0.0583	0.0396	0.0861
σ_L	0.0229	0.0044	0.0474
σ_m	0.0083	0.0054	0.0105
σ_β	0.0723	0.0606	0.0820
σ_θ	0.1329	0.0995	0.1538

Table 9: Posterior Distribution Model 2 (Financial Frictions, No Credit Shocks)

Parameter	Posterior Mean	5th Percentile	95th Percentile
ρ_R	0.5257	0.4466	0.6088
γ_π	1.0328	0.8593	1.2076
γ_y	0.1173	0.0684	0.1647
γ_{q_H}	0.2412	0.1919	0.2935
χ_w	0.7816	0.5931	0.9807
χ_p	0.6136	0.4015	0.7979
ξ_p	0.7215	0.6595	0.7852
ξ_w	0.4910	0.4244	0.5518
ψ_K	0.1367	0.0574	0.2064
ψ_H	0.1342	0.0934	0.1800
ρ_A	0.8980	0.8500	0.9473
ρ_κ	0.8558	0.7799	0.9392
ρ_{ε_H}	0.8142	0.6905	0.9344
ρ_H	0.8222	0.7242	0.9439
ρ_L	0.6756	0.5337	0.8187
ρ_β	0.9781	0.9671	0.9893
ρ_θ	0.4264	0.3194	0.5371
ρ_{ε_R}	0.1128	0.0378	0.1883

Table 10: Posterior Distributions for S.D. (Model 2)

Parameter	Posterior Mean	5th Percentile	95th Percentile
σ_A	0.0106	0.0089	0.0121
σ_κ	0.0036	0.0026	0.0045
σ_R	0.0061	0.0047	0.0075
σ_H	0.0336	0.0099	0.0525
σ_L	0.1980	0.1095	0.2954
σ_β	0.0841	0.0649	0.1027
σ_θ	0.2113	0.1412	0.2761

Table 11: Posterior Distribution for Model 3 (No Financial Frictions)

Parameter	Posterior Mean	5th Percentile	95th Percentile
ρ_R	0.7450	0.7076	0.7758
γ_π	0.9670	0.9650	1.1406
γ_y	0.2108	0.1723	0.2827
γ_{q_H}	0.0949	0.0811	0.1155
χ_w	0.4694	0.4566	0.4748
χ_p	0.5539	0.3190	0.8023
ξ_p	0.8120	0.7592	0.8666
ξ_w	0.9221	0.9177	0.9248
ψ_K	5.8330	4.0336	6.8682
ψ_H	3.8802	3.8105	3.9965
ρ_A	0.9991	0.9990	0.9992
ρ_κ	0.8744	0.8654	0.8858
ρ_{ε_H}	0.5980	0.5977	0.6046
ρ_H	0.5991	0.5986	0.6036
ρ_L	0.9846	0.9845	0.9847
ρ_β	0.9314	0.9307	0.9318
ρ_θ	0.7702	0.7656	0.7713
ρ_{ε_R}	0.0569	0.0568	0.0624

Table 12: Posterior Distributions for S.D. (Model 3)

Parameter	Posterior Mean	5th Percentile	95th Percentile
σ_A	0.0105	0.0097	0.0122
σ_κ	0.0026	0.0024	0.0028
σ_R	0.0031	0.0030	0.0036
σ_H	0.8080	0.7699	0.8850
σ_L	0.8148	0.7454	0.9885
σ_β	0.1134	0.1090	0.1227
σ_θ	0.1374	0.1139	0.1708

Table 13: Model 1 (includes Financial Frictions and Credit Shocks)

Variance Decomposition (in percent)											
	Productivity	Country-Specific Risk Premium	Monetary Policy Shock	Credit Conditions	Housing Demand	Labour Supply	Time Preference	Mark Up	Foreign Output	Foreign Inflation	Foreign Interest Rate
Output	5.37	6.01	2.45	17.50	2.85	0.25	17.20	25.56	16.10	1.34	5.36
Inflation	1.46	6.05	4.40	19.72	1.07	0.06	5.14	44.19	11.42	0.77	5.73
Consumption	1.02	0.28	0.30	4.65	0.67	0.06	83.79	6.50	2.07	0.23	0.44
House Price	0.69	1.05	79.45	8.47	5.75	0.00	0.07	3.89	0.10	0.47	0.05
Mortgage Credit	0.83	0.07	19.92	4.44	1.02	0.03	70.64	0.99	1.38	0.38	0.31
Residential Investment	2.38	1.45	0.85	9.64	14.80	0.10	55.83	13.77	0.89	0.13	0.17

Table 14: Model 2 (Financial Frictions with no Credit Shocks)

Variance Decomposition (in percent)										
	Productivity	Country-Specific Risk Premium	Monetary Policy Shock	Housing Demand	Labour Supply	Time Preference	Mark Up	Foreign Output	Foreign Inflation	Foreign Interest Rate
Output	17.90	3.48	1.22	0.57	31.27	30.20	9.68	3.58	1.16	0.93
Inflation	5.07	6.25	4.23	1.86	12.61	4.02	60.93	2.22	0.24	2.57
Consumption	1.85	0.08	0.07	0.13	2.60	94.18	0.74	0.24	0.09	0.03
House Price	0.74	3.28	65.11	3.55	0.15	0.30	25.71	0.51	0.64	0.01
Mortgage Credit	0.71	0.40	4.35	1.40	1.31	86.64	4.62	0.54	0.01	0.01
Residential Investment	4.10	0.90	0.27	22.83	7.85	58.26	5.33	0.09	0.13	0.24

Table 15: Model 3 (No Financial Frictions and no Credit Shocks)

Variance Decomposition (in percent)										
	Productivity	Country-Specific Risk Premium	Monetary Policy Shock	Housing Demand	Labour Supply	Time Preference	Mark Up	Foreign Output	Foreign Inflation	Foreign Interest Rate
Output	17.71	14.14	4.75	0.01	61.54	0.83	0.88	0.10	0.01	0.03
Inflation	21.79	13.45	2.74	0.01	36.55	4.24	21.19	0.01	0.01	0.03
Consumption	12.45	2.87	1.17	0.00	71.57	11.80	0.12	0.01	0.00	0.01
House Price	16.89	8.11	11.79	34.24	24.07	0.72	4.10	0.04	0.00	0.02
Residential Investment	19.56	8.10	3.14	0.03	66.89	1.78	0.43	0.05	0.00	0.02

Figure 1: Productivity Shock

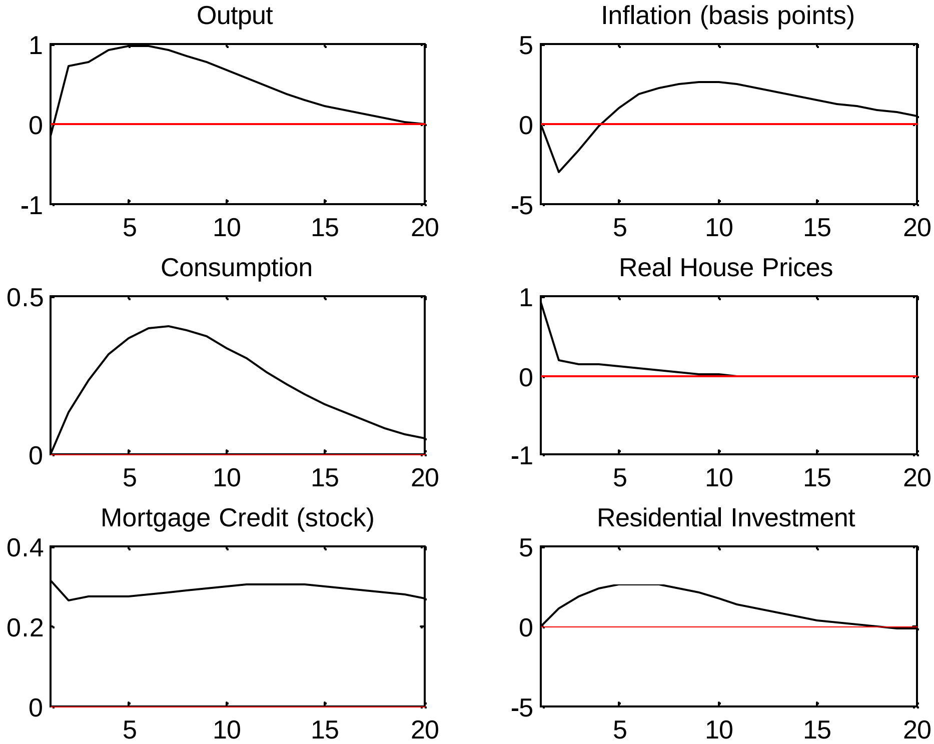

Figure 2: Monetary Policy Shock

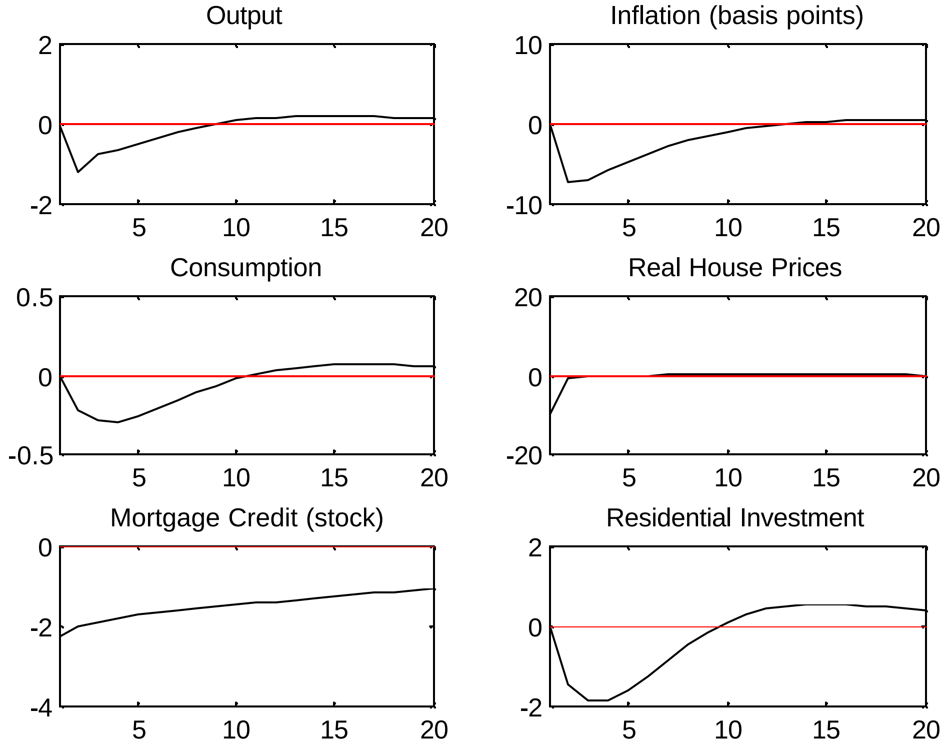

Figure 3: Credit Conditions Shock

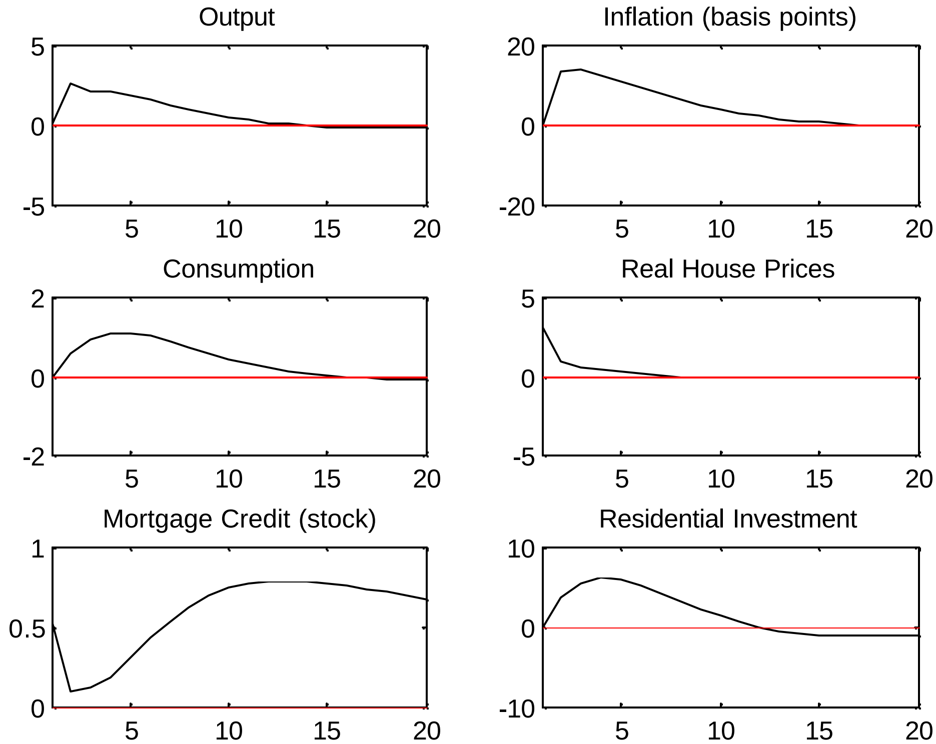

Figure 4: Housing Demand Shock

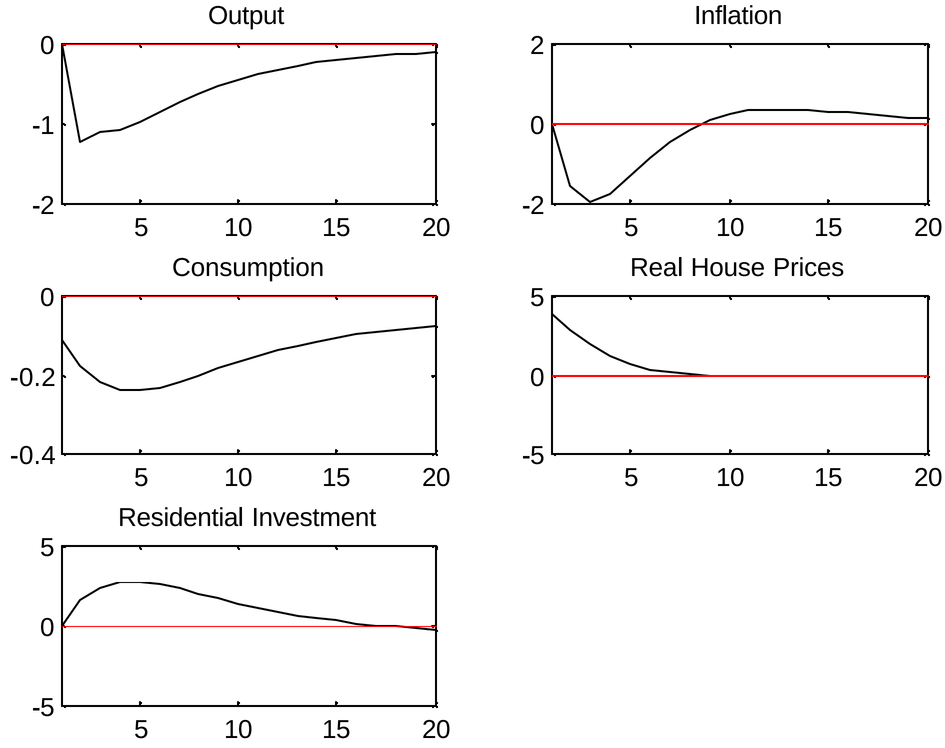

Figure 5: Time Preferences Shock

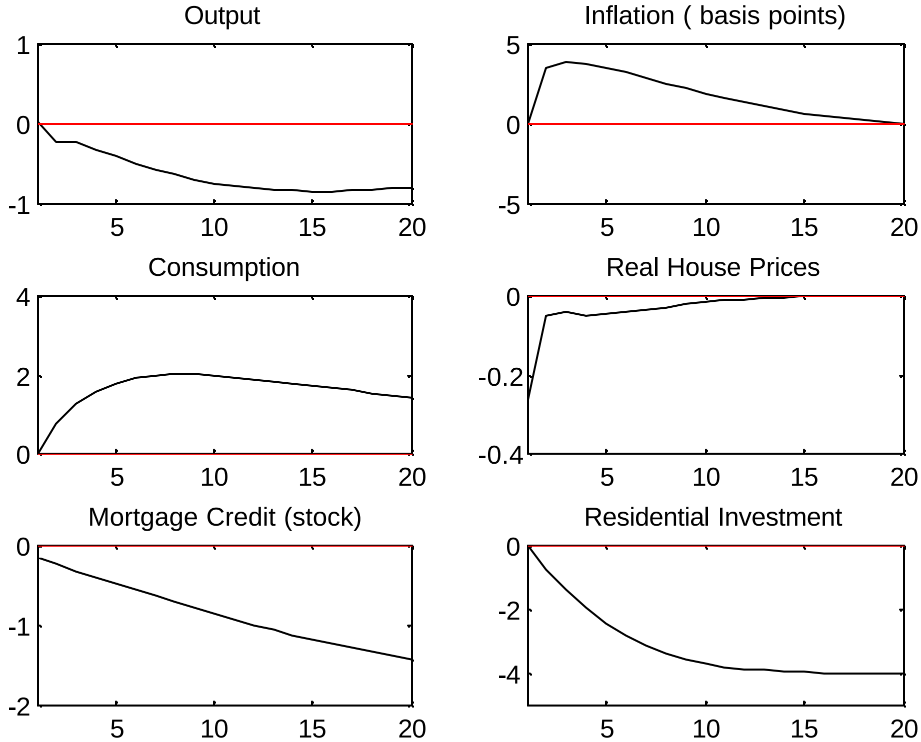

Figure 6: Markup Shock

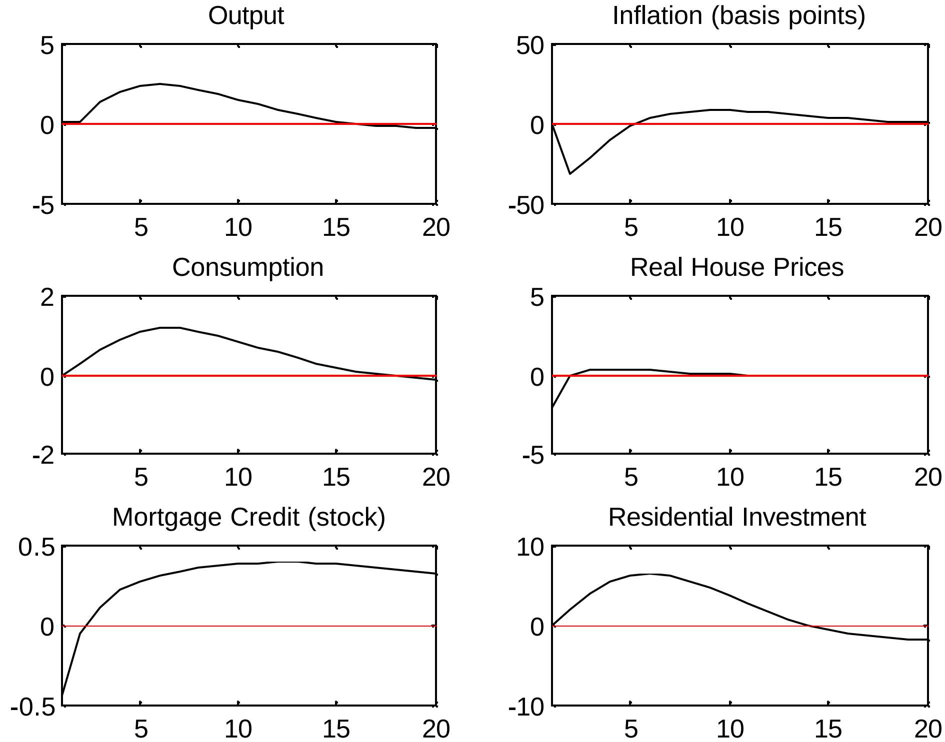

Figure 7: Labour Supply Shock

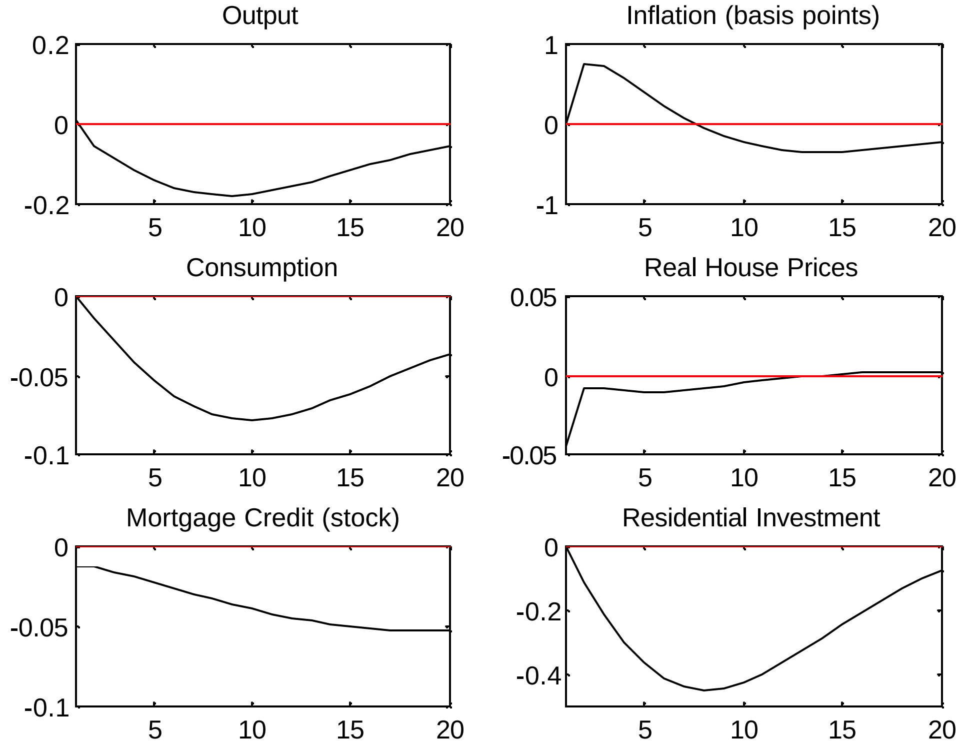

Figure 8: Country-Specific Risk Premium Shock

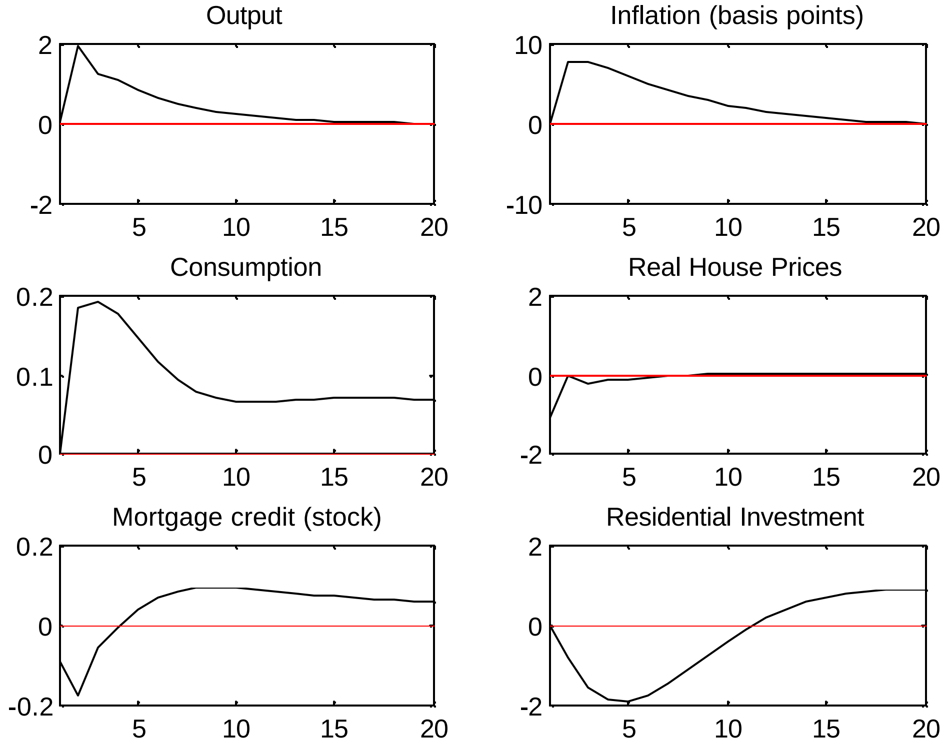

Figure 9: Foreign Output Shock

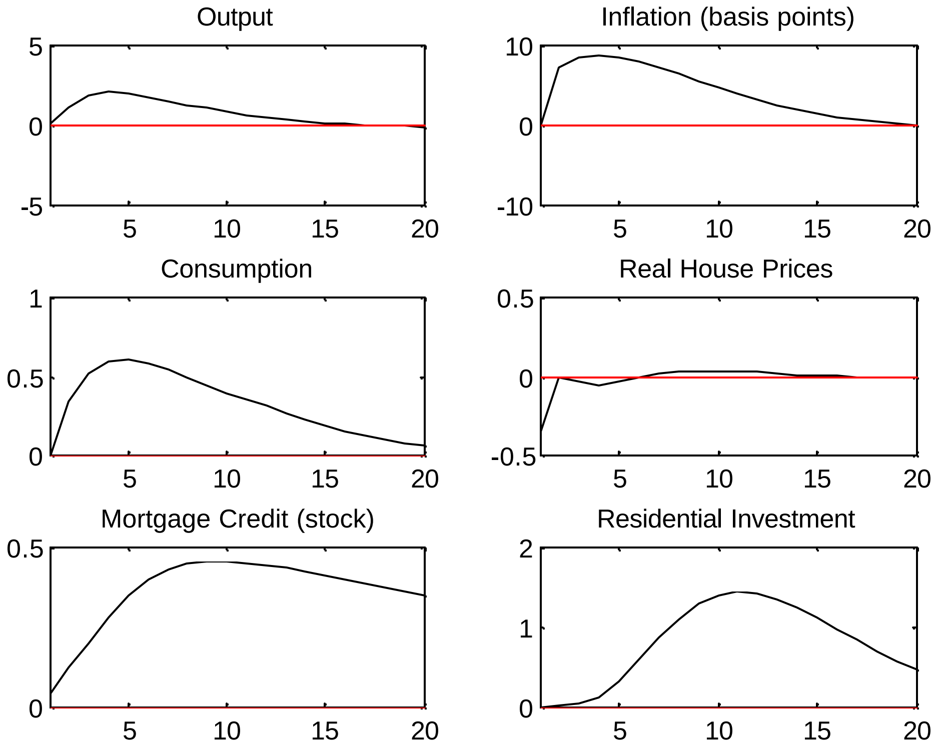

Figure 10: Production