A Portfolio Model of Quantitative Easing
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Legal Issues

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Abstract

This paper presents a portfolio model of asset price effects arising from central bank large-scale asset purchases, commonly known as quantitative easing (QE). Two financial frictions—segmentation of the market for central bank reserves and imperfect asset substitutability—give rise to two distinct portfolio effects. One derives from the reduced supply of the purchased assets. The other runs through banks' portfolio responses to the created reserves and is independent of the assets purchased. The results imply that central bank reserve expansions can affect long-term bond prices even in the absence of long-term bond purchases.

1 Introduction

Since the global financial crisis of 2007-2009, a number of central banks have conducted large-scale asset purchases, often called quantitative easing (QE), in order to provide monetary policy stimulus. Although the stated aims of such purchases have differed across countries, a common objective has been to reduce long-term interest rates, either broadly or in specific markets. In the case of the U.S. Federal Reserve, the success of its QE programs in reducing Treasury yields and mortgage rates appears to be well established; see Gagnon et al. (2011) and Krishnamurthy and Vissing-Jorgensen (2011) among others. Similar evidence for the Bank of England's QE programs is provided in Joyce et al. (2011) and Christensen and Rudebusch (2012). More generally, it is well established that monetary policy can affect long-term interest rates when short-term policy rates are constrained at the effective lower bound (Ball et al. 2016, Swanson and Williams 2014a,b, Wright 2012).

Despite the empirical success of QE, how exactly it helps lower long-term interest rates is not well understood. Research so far has focused on two main channels.[1] One is a signaling channel, which means asset purchases send a signal to investors that lowers market expectations about future monetary policy (Christensen and Rudebusch 2012, Bauer and Rudebusch 2014). If financial market participants perceive that short-term interest rates will be lower in the future, this should translate into lower long-term interest rates today to make investors indifferent between rolling over a short-term loan and committing to a long-term loan. The other channel is a supply-induced portfolio balance channel. When a central bank purchases long-term bonds, it reduces the amount of these bonds available in the market and thereby raises their price and lowers their return (Gagnon et al. 2011).

When a central bank purchases assets as part of a QE program, however, it does not just reduce the supply of these assets in the market. It pays for these assets by issuing new central bank reserves. Hence, the supply of central bank reserves increases one-for-one with the reduction in the purchased assets. Such reserve expansions *per se* may play an important role in the transmission of QE to interest rates, as suggested by Bernanke and Reinhart (2004). Nevertheless, the reserve expansions from QE programs have received little attention in the literature.

A notable exception is Christensen and Krogstrup (2016, henceforth CK), who examine three unique episodes in which the Swiss National Bank expanded reserves by purchasing only short-term debt securities. First, CK document that although the supply of long-term Swiss government bonds and their closest substitutes remained unchanged, long-term yields on benchmark Swiss Confederation bonds fell following the QE announcements. Furthermore, CK show that the fall in rates could not be explained by a lower expected path of short-term

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[1]There are other potential channels for QE to work. For example, it may affect liquidity and market functioning; see Kandrac (2014) and Christensen and Gillan (2016) for discussions and analysis in the context of U.S. QE programs. Also, it may affect the perception and pricing of risk leading to a so-called "risk-taking channel", as discussed in Borio and Zhu (2012).

interest rates, thereby ruling out the signaling channel. Instead, they conclude that the anticipated creation of reserves was responsible for the fall in longer-term yields. Thus, there is empirical evidence that the creation of reserves from QE could matter for its transmission to long-term rates.[2]

The main theoretical reference for QE to give rise to portfolio effects is the model described in Vayanos and Vila (2009), which features preferred habitat behavior among investors, as originally proposed by Modigliani and Sutch (1966, 1967). Unfortunately, this model cannot account for any reserve-induced portfolio effects because it does not contain a central bank or central bank reserves. It models QE transactions as exogenous reductions in the supply of long-term bonds, thereby abstracting from the implications of the increase in central bank reserves for the balance sheets and asset demands of the private sector.[3]

This paper attempts to fill this gap by including a central bank and depository commercial banks in an otherwise simple portfolio model of financial markets, in the spirit of Tobin (1969). In the model, two financial frictions are key to our results. First, assets, including central bank reserves, deposits, and bonds, are imperfect substitutes, and asset substitutability can differ across market participants. Imperfect asset substitutability is also the key friction driving traditional supply-induced portfolio balance effects in models such as Vayanos and Vila (2009). We additionally assume that central bank reserves can only be held by banks (a specific type of market segmentation), while the assets the central bank purchases can be held by banks and nonbank financial firms alike. Both frictions are empirically relevant. Markets for reserves are clearly restricted to certain types of financial institutions that are counterparties to the central bank. Usually, these are depository banks. Moreover, the requirement that asset substitutability is imperfect and can give rise to supply effects on asset prices has ample empirical support (Laubach 2009, Krishnamurthy and Vissing-Jorgensen 2012, Greenwood and Vayanos 2010, 2014, Hamilton and Wu 2012). The nature of the asset substitutability in question has been described in different ways in the literature, e.g., as imperfect substitutability between specific securities leading to local supply effects (D'Amico and King 2013), or as a more broad based imperfect substitutability of duration risk in private portfolios (Gagnon et al. 2011, Li and Wei 2013).

Our model features the traditional portfolio effects as in Vayanos and Vila (2009) arising from the reduced supply of assets available to financial market participants when the central bank conducts QE—a supply-induced portfolio balance effect. Furthermore, it shows that the reserve expansions that accompany QE asset purchases may lead to additional portfolio balance effects on asset prices more broadly. These arise when QE asset purchases are per-

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[2]Another notable exception is Kandrac and Schlusche (2016), who take a step beyond the immediate portfolio balance impact of QE on bond yields and assess the effect of QE-induced reserve accumulations on bank-level lending and risk-taking activity in the U.S. Their results also suggest that the accumulation of reserves per se matters for the transmission of quantitative easing.

[3]See Hamilton and Wu (2012) and Greenwood and Vayanos (2014) for empirical applications of the Vayanos and Vila (2009) model to the U.S. Treasury market.

formed as transactions with nonbank entities. Since they are not banks, they cannot be paid for their assets in reserves. Instead, they are paid in bank deposits by their correspondent banks, and the correspondent banks—and, hence, the banking sector as a whole—see an expansion of their balance sheets with reserves on the asset side and deposits on the liability side. Importantly, banks are passive observers of these transactions that dilute the average risk and return (or duration) of their asset portfolios. To offset this dilution, banks increase their demand for long-term bonds (or other risky assets), which pushes up the prices of these assets further in equilibrium and hence reinforces the supply-induced portfolio balance effect. Such reserve-induced portfolio effects are independent of the particular assets the central bank purchases and arise due to the segmentation of the market for central bank reserves in combination with imperfect asset substitution. Without either of these two financial frictions, the reserve-induced portfolio balance channel highlighted in this paper and empirically investigated in CK shuts down.

Our analysis is related to recent theoretical work on unconventional monetary policy transmission by Farmer and Zabczyk (2016), who demonstrate how, in a general equilibrium overlapping generations model, a change in the risk composition of the central bank balance sheet can affect equilibrium asset prices. Unlike the QE analyzed in this paper, there is no expansion of the central bank balance sheet in their analysis. The authors refer to this type of unconventional monetary policy as qualitative easing. An example of such a program would be the Federal Reserve's Maturity Extension Program (MEP), which operated from September 2011 through December 2012, and changed the composition but not the size of the Federal Reserve's balance sheet. Li and Wei (2013) find that indeed, the MEP appears to have helped push long-term Treasury yields lower, consistent with the model of Farmer and Zabczyk (2016). The key friction generating the results in Farmer and Zabczyk (2016) is caused by incomplete markets for financial contracts across generations, with some yet to be born. Their frictions hence work across time, while ours relate to the opportunity sets across agents. In this sense, the two analyses complement each other.

The remainder of the paper is structured as follows. Section 2 presents the model, while Section 3 investigates the effect of central bank asset purchases on equilibrium asset prices within the model. Section 4 briefly reviews the U.S. experience with QE and assesses whether the model's predictions are consistent with the data. Section 5 concludes and discusses some potential policy implications. The appendix contains an augmented version of the model with two traded assets.

2 The Model

We develop a model of a financial market with three types of agents: banks, nonbank financial firms, and a central bank. The model characterizes asset markets and abstracts from its links to the real economy. Importantly, we assume that over the horizon of the model, banks'

credit portfolios do not adjust. For the same reason, we take the outstanding stock of bonds as given. The purpose of the model is to assess the short-term impact of QE transactions on asset prices before those prices affect real economic outcomes.

The market for tradable securities comprises one asset. Because we have recent QE programs in mind in which long-term bonds have been acquired in exchange for reserves, we refer to this asset as a long-term bond with predetermined supply L and price P_L. In general, however, we can think of it as any traded asset in the economy, including short-term bonds, risky bonds, or equity.

In addition to holding the long-term bond, banks can hold reserves, denoted R, with the central bank and trade them with other banks. A key friction in our model—as in the real world—is that nonbank financial firms cannot hold reserves; instead they hold deposits with their correspondent banks. Another central assumption is that assets are imperfect substitutes, as is common in the portfolio balance literature.

Without any dynamic description in the model, the difference between the price of the long-term bond and its notional value of 1 can be interpreted as capturing the term premium on long-term bonds

$$TP = 1 - P_L. \tag{1}$$

In our model, this term premium arises solely from imperfect substitution between bonds, reserves, and deposits and does not contain any liquidity or credit risk premiums because they are assumed away.

Unlike a deposit, which is on demand, in order to recover the money on a bond, the owner must find a buyer, which requires time and effort and involves some uncertainty about the achievable price. Investors are aware of this, and forward-looking behavior on their part provides the rationale for the existence of the term premium. Within the simplified world of the model, we think of the term premium as being positive, and without it, all agents would prefer to hold deposits.

Having only one traded asset makes the model tractable and suffices to demonstrate the existence of the reserve-induced portfolio balance channel we highlight. However, this is a limitation when it comes to illustrating how central bank asset purchases of one asset can affect the prices of other assets through reserve-induced effects. We therefore consider in the appendix the case with two traded assets, which confirms the findings from the one-asset model and demonstrates the general nature of our findings. Importantly, we show that the reserve-induced portfolio balance channel can affect long-term bond prices even when no long-term bonds are purchased.

To keep the model as simple and as tractable as possible, we analyze the link between central bank asset purchases and asset prices considering a static asset market equilibrium, relying on total differentiation and comparative statics. Specifically, we study how marginal changes to the central bank asset holdings matched by similar changes in the outstanding

amount of central bank reserves affect the equilibrium bond price. There are no dynamics in the model, and when we refer to changes or flows in the following, we are talking about differences between two static equilibria.

2.1 The Central Bank

The balance sheet of the central bank is given by

$$P_L L_{CB} = E_{CB} + R, \tag{2}$$

where L_{CB} is the central bank's holdings of long-term bonds, E_{CB} is the value of the central bank's initial level of equity, and R is the amount of outstanding reserves. Also, there are no cash balances in the model. To study QE, we assume that L_{CB} is the central bank's policy tool, which then determines the level of reserves R as a residual. Equation (2) implies that the change in the central bank's equity can be written as

$$dE_{CB} = dP_L L_{CB} + P_L dL_{CB} - dR. \tag{3}$$

In our simple setup, changes in reserves are matched by changes in the central bank's bond holdings, i.e., $dR = P_L dL_{CB}$. Consequently, changes in the central bank's equity are due solely to changes in the bond price. One practical implication of this is that a central bank engaging in QE is exposed to interest rate risk on its balance sheet; see Christensen et al. (2015) for a detailed discussion along with an empirical assessment for the U.S. Federal Reserve.

2.2 The Nonbank Financial Sector

There is a continuum of nonbank financial firms (e.g., pension funds, money market mutual funds, asset managers, hedge funds etc.) that are fully financed by a predetermined amount of equity. The representative nonbank financial firm holds some combination of bonds and bank deposits as assets. Bank deposits do not pay interest. This simplifying assumption is without much consequence in the kind of environments with near-zero interest rates that typically prevails when central banks launch and operate QE programs. The assets and liabilities of firm j must satisfy

$$P_L L_{NB}^j + D_{NB}^j = E_{NB}^j, \tag{4}$$

where L_{NB}^j is firm j's holdings of bonds, D_{NB}^j is its holdings of bank deposits, and E_{NB}^j is its initial equity value. Changes in the firm's equity are determined as a residual from the flow identity

$$dE_{NB}^j = dP_L L_{NB}^j + P_L dL_{NB}^j + dD_{NB}^j. \tag{5}$$

Firms cannot issue new debt or equity, which is another assumption justified by the short-term nature of the model. Therefore, firms can only obtain deposits by selling assets

$$dD_{NB}^j = -P_L dL_{NB}^j. \tag{6}$$

Equations (5) and (6) imply that changes in firm j's equity value derive from changes in the price of the long-term bond only

$$dE_{NB}^j = dP_L L_{NB}^j. \tag{7}$$

We assume that the firm balances its liquid portfolio between deposits and bonds, demanding positive amounts of both. This portfolio balancing arises because deposits provide liquidity without any return, while bonds generate positive returns, but with less liquidity. Firm j's demand for bonds is, hence, a function of the bond price and its equity

$$L_{NB}^j = f_{NB}(P_L, E_{NB}^j), \tag{8}$$

and it has standard preferences so that its demand is declining in the bond price

$$\frac{\partial f_{NB}(P_L, E_{NB}^j)}{\partial P_L} < 0. \tag{9}$$

Furthermore, even though its equity is determined as a residual by the change in the bond price as stated in equation (7), we assume that the firm will not respond to such equity value changes in real time by changing its demand for bonds, i.e.,

$$\frac{\partial f_{NB}(P_L, E_{NB}^j)}{\partial E_{NB}} = 0. \tag{10}$$

This assumption allows us to abstract from interaction terms that would make the model quite intractable, but it would not be central to the mechanisms we are interested in describing. It now follows that changes in firm j's demand for bonds are purely driven by changes in the bond price

$$dL_{NB}^j = \frac{\partial f_{NB}(P_L, E_{NB}^j)}{\partial P_L} dP_L. \tag{11}$$

Finally, we know from equation (6) that firm j's demand for deposits is a function of the change in its bond holdings. Therefore, its deposits will change with the bond price according to

$$dD_{NB}^j = -P_L \frac{\partial f_{NB}(P_L, E_{NB}^j)}{\partial P_L} dP_L. \tag{12}$$

2.3 The Banking Sector

There is also a continuum of banks. Bank i's assets and liabilities must satisfy

$$R^i + P_L L_B^i = E_B^i + D_B^i, \qquad (13)$$

where L_B^i is bank i's holdings of bonds and R^i is its holdings of central bank reserves. As mentioned earlier, banks' credit portfolios are assumed fixed in the short run and hence, are normalized to zero for simplicity. D_B^i is bank i's deposits from nonbank financial firms. Deposits from nonbanks are endogenously determined, and the bank cannot influence them, given that it does not create new deposits by extending credit or change deposit rates within the horizon considered. We hence define changes in deposits by equation (12), and to keep things simple, we assume symmetry across banks and that there is an identical number of banks and nonbanks. This implies that $D_B^i = D_{NB}^j$. Finally, E_B^i denotes bank i's initial equity level. Over the horizon considered in the model, a bank cannot issue new equity or debt as this is time consuming and requires board approval, etc. Consequently, it can only actively increase its holdings of reserves by selling bonds. On the other hand, reserves can fluctuate autonomously as the bank's customers vary their deposits with the bank. Importantly, banks cannot take actions to change their deposit holdings and therefore consider them as exogenously given. To summarize, we have the following relationship for the change in bank i's reserve holdings

$$dR^i = dD_B^i - P_L dL_B^i. \qquad (14)$$

In general, changes in bank equity are determined as a residual from the flow equivalent of equation (13)

$$dR^i + P_L dL_B^i + L_B^i dP_L = dE_B^i + dD_B^i. \qquad (15)$$

From equation (14), which shows the change in bank i's reserves, it follows that

$$dE_B^i = L_B^i dP_L. \qquad (16)$$

Thus, variation in bank i's equity is solely due to changes in the bond price.

Banks hold bonds and reserves in their liquid asset portfolio and consider them imperfect substitutes. Without loss of generality, we assume that neither reserves nor deposits pay any interest, but the long bond does because $P_L < 1$. In principle, then, an equal increase in deposits and reserves on a bank's balance sheet has no impact on its profitability in this model. Crucially, however, we do assume that banks' business models are such that they do respond to autonomous changes in reserves and deposit funding, with a portfolio balancing motive in mind. Specifically, individual banks aim for a certain balance between risk (or, since we are assuming that the alternative asset to reserves is long bonds, duration mismatch) and return on their liquid asset portfolios. We summarize these considerations in the following

equation for bank i's bond demand

$$L_B^i = f_B(P_L, E_B^i + D_B^i),\qquad(17)$$

where bank i's funding is $F_B^i = E_B^i + D_B^i$.[4]

The demand for reserves is determined as a residual from the demand for bonds given available funding

$$R_B^i = E_B^i + D_B^i - P_L f_B(P_L, E_B^i + D_B^i).\qquad(18)$$

Similar to nonbanks, we assume that banks have standard preferences for bonds and that they exhibit a downward-sloping demand as a function of the bond price

$$\frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial P_L} < 0.\qquad(19)$$

Finally, the response of bank i's demand for bonds to a change in its funding satisfies the restrictions

$$0 < \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B} < 1.\qquad(20)$$

This final restriction is crucial in the model. If the individual bank leaves all autonomous new deposit funding in reserves, i.e., $\frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B} = 0$, we show in Section 3 that the reserve-induced portfolio balance channel shuts down.

The flow equivalent of bank i's bond demand in equation (17) is given by

$$dL_B^i = \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial P_L} dP_L + \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B}(dE_B^i + dD_B^i).\qquad(21)$$

Since banks cannot respond to changes in equity valuations over the short horizon considered in the model, changes in equity valuations are determined as a residual after other changes have taken place, and hence, they are assumed not to affect the bank's demand for bonds. Alternatively, changes in equity valuations can be interpreted as profits that are paid out to shareholders and, therefore, they not available to fund bond purchases. Either way, this implies that

$$\frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B} dE_B^i = 0,\qquad(22)$$

which reduces equation (21) to

$$dL_B^i = \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial P_L} dP_L + \frac{\partial f_B(P_L, E_B^i + D_B^i)}{\partial F_B} dD_B^i.\qquad(23)$$

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[4]Note that equity and deposits are treated as equal, which is without loss of generality for our results.

2.4 Equilibrium

Under the assumption that there is a continuum of banks identical to each other and another continuum of nonbanks also identical to each other, we can use the above equations to characterize the aggregate banking and nonbanking sectors, respectively, by dropping the i and j superscripts. Since we normalize the continuum of institutions in each category to one, we can further use the individual demand equations as characterizing aggregate sectoral demand. The total offered supply of bonds from the three types of market participants must equal their total demand for bonds, while reserves and deposits are determined as a residual.

The market equilibrium is characterized by the bond price that clears demand for bonds and that makes banks' demand for reserves equal the central bank-determined supply given preferences for assets and the total stock of bonds L.

The balance sheets of the banking and nonbanking sectors and, hence, their budget constraints are linked through deposits. We can write the consolidated budget constraint as

$$P_L \left(L - L_{CB} \right) = P_L \left(L_B + L_{NB} \right), \tag{24}$$

where the flow equivalent is

$$dP_L \left(L - L_{CB} \right) + P_L \left(dL - dL_{CB} \right) = dP_L \left(L_B + L_{NB} \right) + P_L \left(dL_B + dL_{NB} \right).$$

3 The Transmission of QE to Bond Prices

In this section, we analyze the effects of central bank bond purchases in exchange for reserves on the balance sheets of banks and nonbanks in order to shed light on the transmission mechanism of such purchases to the bond price. First, we consider the economy with one traded security, as analyzed so far, before we proceed to a brief analysis of the case with two traded securities. A key purpose is to illustrate how the effects depend on the preferences of the private-sector market participants.

3.1 The General Solution with One Traded Security

To arrive at the general solution with one traded security, we first derive the partial derivative of the price change of the long-term bond with respect to central bank bond purchases. To aid intuition on how this expression relates to the asset substitutability of the two types of agents, we consider two special cases. In the first case, nonbanks exhibit very low asset substitutability, and all assets are purchased from banks. In the second case, banks exhibit very low asset substitutability and all assets are acquired from nonbanks. The resulting portfolio balance effects on asset prices differ substantially between these two cases. Finally, we also consider the solution with specific functional forms for the asset demand equations

to further aid intuition.

The specific situation we consider is one in which the central bank increases its reserve liabilities and bond holdings in tandem without changing the total supply of bonds. Thus, the increase in central bank bond holdings must be offset by an identical decline in private-sector holdings

$$dL_{CB} > 0 \quad \text{and} \quad dL = 0 \quad \Rightarrow \quad dL_{CB} = -dL_{NB} - dL_B. \tag{25}$$

Note that these assumptions map in a direct way to the QE programs conducted by major central banks in recent years.

First, we investigate the impact of the change in bond supply and reserves on the price of bonds using the flow equations derived previously. To do so, insert the market aggregate versions of the nonbank bond demand response in equation (11) and the bank bond demand response in equation (23) into equation (25) to obtain

$$dL_{CB} = -\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} dP_L - \frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} dP_L - \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} dD_B. \tag{26}$$

Next, insert the nonbank deposit response in equation (12) to arrive at

$$dL_{CB} = -\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} dP_L - \frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} dP_L + P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} dP_L. \tag{27}$$

Now, the equilibrium bond price response to the central bank bond purchases can be isolated

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} + \frac{\partial f_B(P_L, E_B+D_B)}{\partial P_L} - P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{\partial f_B(P_L, E_B+D_B)}{\partial F_B}}. \tag{28}$$

Equation (28) shows that the reaction of the equilibrium bond price to the central bank bond purchases depends on the sensitivity of market participants' demand for bonds to changes in the bond price. The first two terms in the denominator capture standard supply-induced portfolio balance effects of the central bank bond purchase on the price of bonds that arise from the reduction in the stock of bonds available to the private sector. The third term, however, captures reserve-induced portfolio effects. Note that if the asset price sensitivity of nonbanks' demand for long-term bonds is zero, or if banks do not respond to a change in deposit funding by changing their demand for long-term bonds, the reserve-induced portfolio balance channel shuts down.

To support intuition for the two distinct portfolio balance effects in the special cases investigated below, we also derive how the quantity of deposits and, hence, the size of banks' balance sheets react to the central bank bond purchases. To see this, insert equation (28) into the market aggregate version of the nonbank deposit response in equation (12) to obtain

$$\frac{dD_B}{dL_{CB}} = -P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{dP_L}{dL_{CB}} \tag{29}$$

or, equivalently,

$$\frac{dD_B}{dL_{CB}} = \frac{P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L}}{\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} + \frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} - P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B}}. \tag{30}$$

Below, we discuss in more depth the nature of supply-induced and reserve-induced portfolio balance effects based on these expressions.

3.1.1 Corner Solution with Bond Purchases from Banks

To better describe the standard supply-induced portfolio balance effect, we first consider the extreme case where the bond demand of the nonbank financial sector has zero sensitivity to changes in the bond price, that is, $\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} = 0$.

This implies that the nonbank sector holds a fixed amount of the bond that does not vary with changes in either the supply of the bond or its price. In turn, equation (30) shows that $\frac{dD_B}{dL_{CB}} = 0$. The quantity of bank deposits remains unaffected by the central bank asset purchases.

Banks will simply sell bonds in exchange for reserves. This leaves the size of banks' aggregate balance sheet unchanged, and it leaves nonbank balance sheets unchanged in terms of both size and composition.

Equation (28) shows that in this case, the outright asset purchases would lead to an increase in the bond price equal to

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L}} > 0. \tag{31}$$

This is a pure supply-induced portfolio balance effect that reflects the price increase necessary to make banks willing to substitute away from bonds and into reserves to meet the central bank bond purchases. Although the ultimate underlying cause for the effect is rooted in banks' aversion to holding more reserves at the expense of bonds, we label it a supply-induced portfolio balance effect because it can equally well be viewed as arising from the reduction in the bond supply generated by the QE bond purchases.

3.1.2 Corner Solution with Bond Purchases from Non-Banks

We now consider the alternative extreme, where banks are the ones with price-insensitive demand for bonds, $\frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L} = 0$. In this case, there is no bond price increase that would make banks substitute away from bonds and toward reserves. Assuming that the price sensitivity of nonbanks' demand for bonds is different from zero, the bond purchases of the central bank would induce the nonbank financial sector to be on the selling side. Importantly, these bond sales would result in an autonomous creation of bank deposits matched by an increase in central bank reserves on banks' balance sheets, since the nonbank financial sector

cannot hold reserves. At first, the increase in deposit funding for the banks equals the total amount of bonds purchased by the central bank, i.e., $dD_B = P_L dL_{CB}$. In response to this increase in deposit funding, banks reallocate some of their new reserves toward bonds, as assumed in equation (20). In turn, this puts additional upward pressure on bond prices and gives rise to further bond sales by nonbanks to banks. This will further expand banks' balance sheets with bonds on the asset side and more deposits on the liability side. Thus, the total change in bank deposits in the new equilibrium is

$$\frac{dD_B}{dL_{CB}} = \frac{P_L}{1 - P_L \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B}} > P_L, \tag{32}$$

while the associated equilibrium bond price increase is given by

$$\frac{dP_L}{dL_{CB}} = \frac{-1}{\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} - P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L} \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B}} > 0, \tag{33}$$

where both inequalities follow from $0 < P_L < 1$ and $0 < \frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B} < 1$.

Equation (33) shows that the effect on bond prices from central bank bond purchases with the nonbank financial sector as the counterparty comes from two sources that reinforce each other. The first is the supply-induced portfolio effect that equals the price increase needed to make the nonbank financial sector willing to give up bonds and hold deposits instead. This is captured by the first term in the denominator of equation (33). The other is the reserve-induced portfolio effect that results from the financial friction that only banks can hold reserves. Since banks now have more deposit funding, they will want to reallocate some of it towards bonds according to their preferences, as reflected in $\frac{\partial f_B(P_L, E_B + D_B)}{\partial F_B}$. However, this requires the nonbank sector to be willing to sell additional bonds, which gives rise to the additional weight $P_L \frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L}$. Importantly, because the two terms in the denominator of equation (33) have opposite signs, it follows that the reserve- and supply-induced portfolio effects reinforce each other and make the full effect greater than either in isolation.

From the two corner solutions, it follows that the initial price impact of QE asset purchases will tend to be large whenever $\frac{\partial f_{NB}(P_L, E_{NB})}{\partial P_L}$ and $\frac{\partial f_B(P_L, E_B + D_B)}{\partial P_L}$ are small, i.e., whenever bond demand is price inelastic and investor behavior could be characterized by preferred habitat. On the other hand, when bond demand is very price sensitive and the derivatives above are large for that reason, the price impact will tend to be modest. Accordingly, it will require large amounts of QE bond purchases to have a notable price impact.

3.1.3   A Simple Example

We end this section with a specific example of functional forms for the demand functions that can provide further intuition. Assume that banks' demand for bonds has the following

functional form consistent with the restrictions laid out in Section 2

$$f_B(P_L, E_B + D_B) = \frac{1 - P_L}{P_L}(E_B + D_B) = \left(\frac{1}{P_L} - 1\right)(E_B + D_B), \qquad (34)$$

where we restrict the possible values of the bond price to $P_L \in (\frac{1}{2}, 1)$. This ensures that the bond demand is smaller than the available funding, $E_B + D_B$, and that no asset positions taken by banks can be negative.

Assume also that the demand for bonds by the nonbank financial sector has the following functional form

$$f_{NB}(P_L, E_{NB}) = E_{NB} - \alpha P_L, \qquad (35)$$

where we impose the restriction that $0 < \alpha < E_{NB}$ to ensure that the bond demand is smaller than the available equity for similar reasons as above.

Now, if we calculate the relevant derivatives with respect to the bond price and bank deposits and insert the results in equation (28), we obtain the effect of central bank bond purchases on the price of the bond

$$\frac{dP_L}{dL_{CB}} = \frac{1}{\alpha + \frac{E_B + D_B}{P_L^2} - \alpha(1 - P_L)}. \qquad (36)$$

The first term in the denominator is the price sensitivity of the demand for bonds by nonbanks. The second term in the denominator is the sensitivity of banks' demand for bonds to a change in their price. The greater these two price sensitivities, the smaller the increase in the price of bonds due to supply-induced portfolio balance effects. The last term represents the reserve-induced portfolio balance effect. This term is an interaction between the price sensitivity of the bond demand by nonbanks and the sensitivity of banks' bond demand to changes in deposit funding. Under our functional assumptions, the reserve-induced portfolio balance effect will tend to be smaller as the term premium falls. In the case of a zero term premium $(1 - P_L = 0)$, there is no reserve-induced portfolio balance effect on bond prices from central bank asset purchases, since, in this case, banks will not increase their demand for bonds in response to an increase in deposit funding.

3.2 The Case of Two Traded Securities

To demonstrate that supply- and reserve-induced portfolio balance effects are distinct, we now consider the case with two traded securities, a short bond S and a long bond L. We note that, for the U.S. and the U.K., the relevant case is one in which long-term bonds are purchased as discussed so far. However, to demonstrate that the reserve-induced portfolio balance channel can affect long-term bond prices without any purchases of long-term bonds as argued in CK, we need to study the augmented model with two traded securities. The full model is described in the appendix and briefly summarized in the following.

To map to the analysis in CK, we consider a situation in which the central bank is implementing a QE program through purchases of short bonds only, i.e., $dS_{CB} > 0$ and $dL_{CB} = 0$.

Using notation similar to that introduced in Section 2, banks' demand for short and long bonds is given by

$$S_B = f_B^S(P_S, P_L, E_B + D_B) \quad \text{and} \quad L_B = f_B^L(P_S, P_L, E_B + D_B),$$

while the corresponding demand functions of the nonbank institutions are

$$S_{NB} = f_{NB}^S(P_S, P_L, E_{NB}) \quad \text{and} \quad L_{NB} = f_{NB}^L(P_S, P_L, E_{NB}).$$

Calculations provided in the appendix show that the equilibrium response of the long bond price to short bond purchases by the central bank is, in general, given by

$$\frac{dP_L}{dS_{CB}} = -\frac{\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_S} + P_L \frac{\partial f_{NB}^L}{\partial P_S})}{\Delta}, \tag{37}$$

where

$$\begin{aligned}
\Delta &= \left(\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_S} + P_L \frac{\partial f_{NB}^L}{\partial P_S})\right)\left(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L}\right) \\
&\quad - \left(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_L} + P_L \frac{\partial f_{NB}^L}{\partial P_L})\right)\left(\frac{\partial f_{NB}^S}{\partial P_S} + \frac{\partial f_B^S}{\partial P_S}\right).
\end{aligned}$$

This is a complex expression, and we impose a number of simplifying assumptions to make our point clear. First, we assume that the demand for short bonds by nonbank financial institutions is characterized by perfect price elasticity, i.e., $\frac{\partial f_{NB}^S}{\partial P_S} \to \infty$. Also, we assume that banks do not vary their demand for short bonds in response to changes in the short bond price, i.e., $\frac{\partial f_B^S}{\partial P_S} = 0$. These assumptions ensure that the central bank purchases will be performed with nonbank entities as counterparties, which allows us to demonstrate the reserve-induced portfolio balance effect most clearly, but we stress that these assumptions are not necessary to ensure its existence. Combined, the two assumptions reduce equation (37) to

$$\frac{dP_L}{dS_{CB}} = \frac{\frac{\partial f_B^L}{\partial F_B} P_S}{-\frac{\partial f_B^L}{\partial F_B} P_S(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L}) - \left(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_L} + P_L \frac{\partial f_{NB}^L}{\partial P_L})\right)}. \tag{38}$$

Second, we assume that all cross-price elasticities are zero, say, $\frac{\partial f_B^S}{\partial P_L} = 0$, i.e., we are in

the extreme case of no substitution effects between assets. This reduces equation (38) to

$$\frac{dP_L}{dS_{CB}} = \frac{-\frac{\partial f_B^L}{\partial F_B} P_S}{\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - P_L \frac{\partial f_{NB}^L}{\partial P_L} \frac{\partial f_B^L}{\partial F_B}} > 0. \tag{39}$$

Since we assume standard preferences for banks and nonbanks, $\frac{\partial f_{NB}^L}{\partial P_L} < 0$ and $\frac{\partial f_B^L}{\partial P_L} < 0$, and we continue to assume that banks respond to changes in their funding conditions by balancing their portfolios, i.e., $\frac{\partial f_B^L}{\partial F_B} \in (0,1)$, it follows that the long bond price response to QE short bond purchases in equation (39) is positive.

This shows that the price of long bonds can be positively affected when the central bank engages in quantitative easing by buying short-term bonds, as was the case in Switzerland in August 2011. This makes the point that in a general setting with multiple securities, supply- and reserve-induced portfolio balance effects are two separate transmission channels for QE asset purchases to affect long-term interest rates.

Specifically, when a central bank implements a QE program by purchasing short-term bonds, there is no supply-induced portfolio balance effect on long bond prices. However, reserve-induced portfolio balance effects can continue to exist, provided the QE transactions are performed with nonbank financial institutions and run through banks' portfolio responses to the created reserves.

This effect does not come from cross-price demand elasticities as we have fixed those at zero. Instead, the effect comes from banks' increased demand for long bonds in response to the associated expansion of their balance sheets. Fixing the cross-price elasticities to zero is not necessary to obtain this result. It remains valid as long as the cross-price elasticities are smaller in absolute value than the direct own-price elasticities.

Finally, it remains the case that supply- and reserve-induced portfolio effects reinforce each other. When banks purchase long bonds from nonbanks, they pay with deposits and, hence, expand their balance sheets beyond what the QE purchases by themselves would imply.

4 Empirical Support for Reserve-Induced Portfolio Effects

The portfolio model analyzed in the previous sections predicts that if central bank asset purchases are conducted in short-term assets only, standard supply-induced portfolio balance effects would not materialize, and bond risk premiums would only be affected provided reserve-induced portfolio balance effects are operating. CK present evidence for exactly such reserve-induced portfolio balance effects on long-term bond prices around three announcements of central bank reserve expansions in Switzerland in August 2011. These expansions were achieved without any purchases of long-term securities.

When the central bank is buying long-term bonds, however, both supply-induced and reserve-induced portfolio balance effects can affect bond risk premiums. The QE programs

of the Federal Reserve, the Bank of England, the Bank of Japan, and the ECB all included purchases of long-term securities. Therefore, the many event studies of the financial market reactions to the announcements of these programs cannot identify supply- and reserve-induced effects separately. Despite this inability to distinguish between supply- and reserve-induced effects directly, it is still possible to empirically verify whether the necessary conditions for reserve-induced portfolio balance effects to exist are satisfied. When central bank asset purchases are conducted with a range of financial intermediaries that includes an important share of nonbank entities, commercial bank balance sheets expand, and the possibility of reserve-induced portfolio balance effects arises. The mix of counterparties to central bank asset purchases is likely to differ across countries and over time, and a fully fledged cross-country econometric analysis of counterparties and bank balance sheets is beyond the scope of this paper. Instead, as a case study, we take a closer look at the literature and available data for the QE programs conducted by the Federal Reserve since the global financial crisis.

As we explain below, the evidence suggests that the Federal Reserve's later QE programs in particular were conducted mainly with nonbanks, that bank balance sheets increased in connection with the asset purchases, and that banks did indeed respond to these changes in their portfolio compositions. There is hence ample scope for reserve-induced portfolio balance effects to have played a central role in the transmission of these QE programs to the prices of long-dated securities.

The question about the counterparties to the Federal Reserve's QE programs is addressed in Carpenter et al. (2015). They analyze data on U.S. financial flows of funds and find that the Fed's purchases are mainly associated with reductions in the holdings of the targeted types of assets by nonbank entities, predominantly households, hedge funds, broker-dealers, and insurance companies.[5]

Ennis and Wolman (2015) study the reaction of individual U.S. commercial bank balance sheets to the increase in reserves during the Federal Reserve's first and second QE programs. Their data suggest that banks played a central role as counterparties during the first QE program (QE1) in 2009, which would limit the role of reserve-induced effects at that time. Thus, banks' securities portfolios fell significantly in response to the increase in reserves during this episode, while there are few signs of changes in bank liabilities. This was a period of financial market stress that affected banks in particular, and QE1 may have helped banks deleverage in an orderly way. In contrast, during the second QE program (QE2) from November 2010 to June 2011, commercial bank deposits were positively associated with increases in reserves, while commercial bank holdings of securities did not respond. Thus, bank balance sheets expanded. The transmission of QE2 is hence likely to have worked very differently from the transmission of QE1, and reserve-induced effects could have been central.

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[5]In the case of the U.K., Joyce et al. (2011) describe how the Bank of England's asset purchase programs were initially conducted in assets held by nonbank financial institutions with the stated intention of boosting broader monetary aggregates.
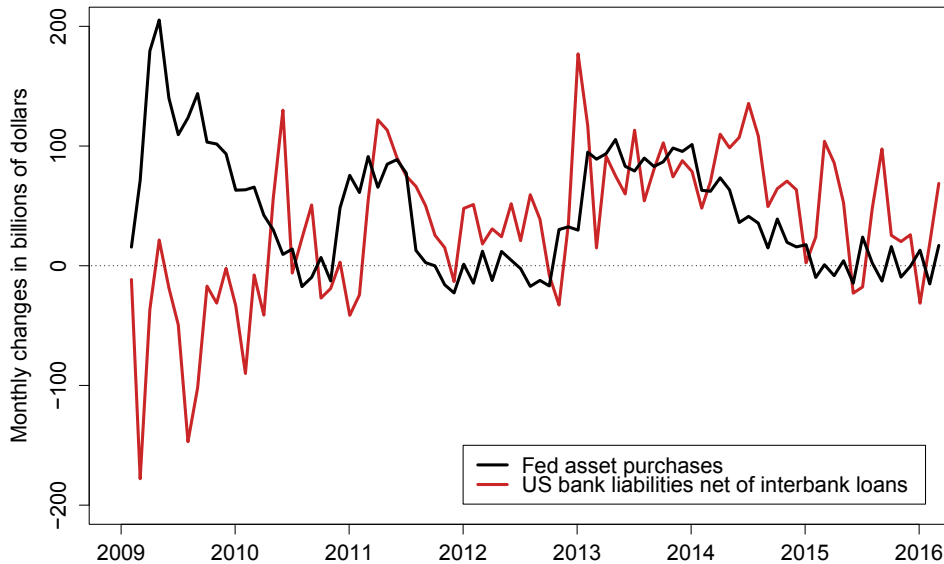
Figure 1: **Fed Asset Purchases and U.S. Bank Liabilities.**
Monthly Federal Reserve asset purchases are approximated using changes in the value of the Federal Reserves' holdings of Treasury and agency securities over the month measured in billions of dollars. Changes in U.S. bank liabilities are measured as the two-month moving average change in total bank liabilities less borrowing from U.S. banks, also measured in billions of dollars. Sources: Federal Reserve, SOMA accounts, and FRB.H8.

Unfortunately, the analysis of Ennis and Wolman (2015) does not extend to the Federal Reserve's third QE program (QE3) that operated from September 2012 to October 2014. To gain further insight, Figure 1 plots the monthly changes in the Federal Reserve's holdings of Treasury securities and mortgage-backed securities as a proxy for monthly central bank asset purchases and the (slightly smoothed) monthly changes in bank balance sheets net of interbank positions from January 2009 to February 2016. The figure confirms the findings of Ennis and Wolman (2015) for QE1 and QE2. The two series move in opposite directions in 2009 but comove strongly following the launch of QE2 in late 2010. The third wave of asset purchases during QE3 is, moreover, characterized by consistent increases in bank balance sheets very similar to the size of the asset purchases. This association is highly statistically significant. Thus, this evidence is consistent with the findings reported in Carpenter et al. (2015) and suggests that the conditions necessary for reserve-induced portfolio balance effects to exist were likely met for the Fed's QE2 and QE3 programs.

Finally, Kandrac and Schlusche (2016) investigate banks' reactions to increases in their reserves holdings during all three QE programs, and they find that banks increased risk taking and expanded their loan portfolios. Thus, their analysis suggests that banks indeed have a

portfolio response to increases in their reserves holdings, which is the other key assumption necessary in our theoretical portfolio model for reserve-induced portfolio balance effects to exist.

Overall, we find that the conditions necessary for reserve-induced portfolio balance effects to have played a role in the financial market response to the Fed's QE programs are likely satisfied. The extent to which they have mattered for the transmission of QE in the U.S. is ultimately an empirical question that we leave for future research.

5 Conclusion

In this paper, we augment a standard portfolio model to include a central bank that issues reserves in exchange for bonds and a banking sector that holds reserves and bonds financed with deposits and equity. The model contains two key frictions. First, reserves can only be held by banks. Second, central bank reserves, bank deposits, and traded securities are imperfect substitutes for each other. We use this model to study how central bank asset purchases affect the behaviors of banks and nonbank financial institutions and their implications for equilibrium asset prices.

We find that provided a share of the central bank asset purchases are performed with nonbanks, they can give rise to two separate portfolio effects on bond prices that reinforce each other. One is due to the reduction in the available supply of bonds—a supply-induced portfolio effect. The other arises from the expansion of reserves that only banks can hold. This friction expands banks' balance sheets, dilutes the duration of their portfolios, and makes them increase their demand for bonds—a reserve-induced portfolio balance effect. In contrast, when preferences of banks and nonbanks are such that the central bank asset purchases are mainly executed with banks, only supply-induced portfolio effects materialize because there is no expansion of banks' balance sheets.

The model gives a theoretical characterization of the circumstances under which reserve-induced portfolio effects can exist. Reviewing recent research and data, we find that these circumstances are likely to have been met for recent Federal Reserve QE programs. Unfortunately, the data do not allow us to empirically identify the importance of reserve-induced effects relative to supply-induced effects in contributing to the transmission of QE to asset prices. Hence, we leave this important question for future research.

More generally, the model suggests that financial market structure, the business models of financial market intermediaries, their portfolio optimizing tools, and bank regulations may affect the transmission of QE to long-term interest rates. This is because these factors affect the substitutability between short- and long-term assets in the portfolios of banks and nonbank entities. Thus, a promising avenue of future research could be to analyze the implications of bank regulation such as the enforcement of leverage ratios for the transmission of QE to asset prices.

Finally, the existence of a reserve-induced portfolio balance channel may also have implications for monetary policy itself. For example, at the operational level, QE programs may be effective at lowering long-term bond interest rates even in the absence of long-term bond purchases. This could matter for central banks that have to operate in markets with a limited supply of long-term securities of sufficiently high credit quality.

Appendix: A Portfolio Model with Two Traded Securities

In this appendix, we present an extension of the baseline model considered in the main text with two traded securities in addition to central bank reserves and bank deposits. To keep the exposition simple, we go straight to the aggregate market equations and ignore the i and j superscripts.

The Central Bank

The central bank balance sheet is now given by

$$P_S S_{CB} + P_L L_{CB} = E_{CB} + R, \tag{40}$$

where S_{CB} is the central bank's holdings of short-term bonds. The change in the equity value of the central bank is given by

$$dE_{CB} = dP_S S_{CB} + P_S dS_{CB} + dP_L L_{CB} + P_L dL_{CB} - dR. \tag{41}$$

We now assume that S_{CB} is the central bank's policy tool and that it determines R, keeping its long-term bond holdings constant, i.e., $dL_{CB} = 0$.

The Nonbank Financial Sector

The aggregate balance sheet of the nonbank financial sector is characterized by

$$P_S S_{NB} + P_L L_{NB} + D_{NB} = E_{NB}, \tag{42}$$

where S_{NB} represents the short-term bond holdings of the nonbank financial sector.

Changes in its equity position are determined as a residual from the flow identity

$$dE_{NB} = dP_S S_{NB} + P_S dS_{NB} + dP_L L_{NB} + P_L dL_{NB} + dD_{NB}. \tag{43}$$

When it sells assets, the nonbank financial sector obtains deposits

$$P_S dS_{NB} + P_L dL_{NB} = -dD_{NB}. \tag{44}$$

By implication, changes in its equity value derive from changes in the prices of its bond holdings

$$dE_{NB} = dP_S S_{NB} + dP_L L_{NB}. \tag{45}$$

The demand for bonds by the nonbank financial sector is a function of the bond prices and equity

$$S_{NB} = f_{NB}^S(P_S, P_L, E_{NB}), \tag{46}$$

$$L_{NB} = f_{NB}^L(P_S, P_L, E_{NB}), \tag{47}$$

while the amount of demand deposits is determined as a residual from the budget constraint and is given in equation (44).

We assume standard preferences with negative own-price effects

$$\frac{\partial f_{NB}^S}{\partial P_S} < 0 \quad \text{and} \quad \frac{\partial f_{NB}^L}{\partial P_L} < 0 \tag{48}$$

and positive cross-price effects

$$\frac{\partial f_{NB}^S}{\partial P_L} > 0 \quad \text{and} \quad \frac{\partial f_{NB}^L}{\partial P_S} > 0, \tag{49}$$

so that short and long bonds are imperfect substitutes.

Furthermore, even though the equity of the nonbank financial sector is determined as a residual by the

change in the bond prices as stated in equation (45), we assume that it will not respond to such equity value changes in real time by changing its demand for bonds, i.e.,

$$\frac{\partial f_{NB}^S}{\partial E_{NB}} = \frac{\partial f_{NB}^L}{\partial E_{NB}} = 0. \tag{50}$$

Combining this with equations (46) and (47), we obtain the bond demand response of the nonbank financial sector to changes in the bond prices

$$dS_{NB} = \frac{\partial f_{NB}^S}{\partial P_S}dP_S + \frac{\partial f_{NB}^S}{\partial P_L}dP_L, \tag{51}$$

$$dL_{NB} = \frac{\partial f_{NB}^L}{\partial P_S}dP_S + \frac{\partial f_{NB}^L}{\partial P_L}dP_L. \tag{52}$$

From equation (44), it then follows that the change in deposit holdings as a consequence of bond price changes is given by

$$dD_{NB} = -P_S\Big(\frac{\partial f_{NB}^S}{\partial P_S}dP_S + \frac{\partial f_{NB}^S}{\partial P_L}dP_L\Big) - P_L\Big(\frac{\partial f_{NB}^L}{\partial P_S}dP_S + \frac{\partial f_{NB}^L}{\partial P_L}dP_L\Big). \tag{53}$$

The Banking Sector

There is a continuum of banks in the economy, and their aggregate balance sheet is characterized by

$$R + P_S S_B + P_L L_B = E_B + D_B, \tag{54}$$

where S_B represents banks' holdings of short bonds, L_B is their holdings of long-term bonds, and R is their reserves, while D_B and E_B represent their deposits and equity, respectively. As in the main text, deposits are endogenously determined by the transactions of the nonbank financial sector.

Note also that we continue to assume that, over the horizon considered in the model, a bank cannot issue new equity or debt as this is time consuming and requires board approval etc. Consequently, it can only actively increase its holdings of reserves by selling bonds. On the other hand, reserves can fluctuate autonomously as the bank's customers vary their deposits with the bank. Importantly, banks cannot take actions to change their deposit holdings. Banks consider them as exogenously given. To summarize, we have the following relationship for the change in banks' reserve holdings

$$dR = dD_B - P_S dS_B - P_L dL_B. \tag{55}$$

Changes in bank equity are determined as a residual from the flow equation (54)

$$dE_B = P_S dS_B + P_S dS_B + L_B dP_L + dR - dD_B.$$

Banks hold long and short bonds and reserves in their liquid asset portfolios. While banks consider short and long bonds to be imperfect substitutes, we assume that they see short bonds and reserves as near-perfect substitutes when the yield of short bonds is near zero, i.e., near the zero lower bound.

Formally, banks' demand for short and long bonds is given by

$$S_B = f_B^S(P_S, P_L, E_B + D_B), \tag{56}$$

$$L_B = f_B^L(P_S, P_L, E_B + D_B). \tag{57}$$

Again, we assume banks have standard preferences for both types of bonds with negative own-price effects

$$\frac{\partial f_B^S}{\partial P_S} < 0 \quad \text{and} \quad \frac{\partial f_B^L}{\partial P_L} < 0 \tag{58}$$

and positive cross-price effects

$$\frac{\partial f_B^S}{\partial P_L} > 0 \quad \text{and} \quad \frac{\partial f_B^L}{\partial P_S} > 0,$$ (59)

so that short and long bonds are imperfect substitutes, as intended.

Finally, the demand for reserves is determined as a residual from the demand for bonds given the available initial equity funding and deposits.

As in the main text, it is key for the existence of the reserve-induced portfolio balance channel that changes in available funding lead to increased demand for short and long bonds

$$0 < \frac{\partial f_B^S}{\partial F_B} < 1 \quad \text{and} \quad 0 < \frac{\partial f_B^L}{\partial F_B} < 1.$$ (60)

These bond demand sensitivities to changes in deposit funding are crucial for our results. Specifically, demand for long bonds is positive, as banks that receive increased deposit funding seek to convert some of the additional liquidity into assets with positive duration.

Since banks cannot respond to changes in equity valuations over the short horizon considered in the model, changes in equity valuations are determined as a residual after other changes have taken place, and hence, they are assumed not to affect the bank's demand for bonds. Alternatively, changes in equity valuations can be interpreted as profits that are paid out to shareholders and are therefore not available to fund bond purchases. Either way, this implies that

$$\frac{\partial f_B^S(P_S, P_L, E_B + D_B)}{\partial F_B} dE_B = 0 \quad \text{and} \quad \frac{\partial f_B^L(P_S, P_L, E_B + D_B)}{\partial F_B} dE_B = 0.$$ (61)

This reduces the flow equivalents of banks' demand for short and long bonds in equations (56) and (57) to

$$dS_B = \frac{\partial f_B^S}{\partial P_S} dP_S + \frac{\partial f_B^S}{\partial P_L} dP_L + \frac{\partial f_B^S}{\partial F_B} dD_B,$$ (62)

$$dL_B = \frac{\partial f_B^L}{\partial P_S} dP_S + \frac{\partial f_B^L}{\partial P_L} dP_L + \frac{\partial f_B^L}{\partial F_B} dD_B.$$ (63)

The Market Equilibrium

The equilibrium in the model is characterized by a set of bond prices and a set of bond allocations across the private sector agents that ensure that the markets for short and long bonds are in equilibrium. Compared to the model with one traded security considered in the main text, we now have interaction terms due to the substitutability between short and long bonds and between bonds and reserves and deposits. This substantially complicates the derivation of the market equilibrium. Therefore, we first present the equilibrium expressions. Second, we make several assumptions we think are appropriate in order to capture the zero lower bound environment in which QE is usually performed. We then evaluate the model implications under those assumptions.

To begin, equilibrium in bond markets requires that

$$dS_{NB} + dS_B + dS_{CB} = 0,$$ (64)

$$dL_{NB} + dL_B + dL_{CB} = 0.$$ (65)

In the following, to highlight the existence of the reserve-induced portfolio balance channel, we consider the case when the central bank implements a QE program by buying short bonds only, i.e., $dS_{CB} > 0$ and $dL_{CB} = 0$. Inserting the flow demand equations of banks and nonbanks, including demand due to changes in nonbanks' demand for deposits, yields the two market clearing conditions.

First, use equations (52), (63), and (53) to determine how the demand for long bonds responds to changes in the relative asset prices

$$dL_{NB} + dL_B = \frac{\partial f_{NB}^L}{\partial P_S}dP_S + \frac{\partial f_{NB}^L}{\partial P_L}dP_L + \frac{\partial f_B^L}{\partial P_S}dP_S + \frac{\partial f_B^L}{\partial P_L}dP_L + \frac{\partial f_B^L}{\partial F_B}dD_B \tag{66}$$

$$= \frac{\partial f_{NB}^L}{\partial P_S}dP_S + \frac{\partial f_{NB}^L}{\partial P_L}dP_L + \frac{\partial f_B^L}{\partial P_S}dP_S + \frac{\partial f_B^L}{\partial P_L}dP_L \tag{67}$$

$$+ \frac{\partial f_B^L}{\partial F_B}\Big(- P_S(\frac{\partial f_{NB}^S}{\partial P_S}dP_S + \frac{\partial f_{NB}^S}{\partial P_L}dP_L) - P_L(\frac{\partial f_{NB}^L}{\partial P_S}dP_S + \frac{\partial f_{NB}^L}{\partial P_L}dP_L)\Big)$$

$$= 0.$$

This can be rearranged to yield

$$dP_S\Big(\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_S} + P_L\frac{\partial f_{NB}^L}{\partial P_S})\Big) + dP_L\Big(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_L} + P_L\frac{\partial f_{NB}^L}{\partial P_L})\Big) = 0. \tag{68}$$

Second, use equations (51), (62), and (53) to determine how the demand for short bonds has to balance out purchases by the central bank and the resulting relative asset price changes

$$dS_{NB} + dS_B = \frac{\partial f_{NB}^S}{\partial P_S}dP_S + \frac{\partial f_{NB}^S}{\partial P_L}dP_L + \frac{\partial f_B^S}{\partial P_S}dP_S + \frac{\partial f_B^S}{\partial P_L}dP_L + \frac{\partial f_B^S}{\partial F_B}dD_B \tag{69}$$

$$= \frac{\partial f_{NB}^S}{\partial P_S}dP_S + \frac{\partial f_{NB}^S}{\partial P_L}dP_L + \frac{\partial f_B^S}{\partial P_S}dP_S + \frac{\partial f_B^S}{\partial P_L}dP_L \tag{70}$$

$$+ \frac{\partial f_B^S}{\partial F_B}\Big(- P_S(\frac{\partial f_{NB}^S}{\partial P_S}dP_S + \frac{\partial f_{NB}^S}{\partial P_L}dP_L) - P_L(\frac{\partial f_{NB}^L}{\partial P_S}dP_S + \frac{\partial f_{NB}^L}{\partial P_L}dP_L)\Big)$$

$$= -dS_{CB}.$$

This can be rearranged to yield

$$dP_S\Big(\frac{\partial f_{NB}^S}{\partial P_S} + \frac{\partial f_B^S}{\partial P_S} - \frac{\partial f_B^S}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_S} + P_L\frac{\partial f_{NB}^L}{\partial P_S})\Big) + dP_L\Big(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L} - \frac{\partial f_B^S}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_L} + P_L\frac{\partial f_{NB}^L}{\partial P_L})\Big) = -dS_{CB}. \tag{71}$$

The two market equilibrium conditions give us two equations with two unknowns, namely, the bond price responses to the central bank short bond purchases ($\frac{dP_L}{dS_{CB}}$, $\frac{dP_S}{dS_{CB}}$) that we are interested in solving for. This system can be written in matrix form as

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}\begin{pmatrix} dP_S \\ dP_L \end{pmatrix} = \begin{pmatrix} 0 \\ -dS_{CB} \end{pmatrix}, \tag{72}$$

where

- $\alpha = \frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_S} + P_L\frac{\partial f_{NB}^L}{\partial P_S})$;
- $\beta = \frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_L} + P_L\frac{\partial f_{NB}^L}{\partial P_L})$;
- $\gamma = \frac{\partial f_{NB}^S}{\partial P_S} + \frac{\partial f_B^S}{\partial P_S} - \frac{\partial f_B^S}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_S} + P_L\frac{\partial f_{NB}^L}{\partial P_S})$;
- $\delta = \frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L} - \frac{\partial f_B^S}{\partial F_B}(P_S\frac{\partial f_{NB}^S}{\partial P_L} + P_L\frac{\partial f_{NB}^L}{\partial P_L})$.

Solving for the two bond price changes, we have that

$$\begin{pmatrix} dP_S \\ dP_L \end{pmatrix} = \frac{1}{\alpha\delta - \beta\gamma}\begin{pmatrix} \delta & -\beta \\ -\gamma & \alpha \end{pmatrix}\begin{pmatrix} 0 \\ -dS_{CB} \end{pmatrix} = \begin{pmatrix} \frac{\beta dS_{CB}}{\alpha\delta - \beta\gamma} \\ \frac{-\alpha dS_{CB}}{\alpha\delta - \beta\gamma} \end{pmatrix}. \tag{73}$$

Thus, the general result is

$$\frac{dP_S}{dS_{CB}} = \frac{\beta}{\alpha\delta - \beta\gamma} \quad \text{and} \quad \frac{dP_L}{dS_{CB}} = -\frac{\alpha}{\alpha\delta - \beta\gamma}. \tag{74}$$

However, these are very complicated expressions, so to help build intuition, we now focus on the case when short-term interest rates are near the zero lower bound. Consequently, we assume the short bond price to be 1, and we assume that the demand for short bonds is characterized by perfect own-price elasticity in

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the neighborhood of $P_S = 1$. We operationalize this by assuming that either banks, nonbanks, or both have demand for short bonds with perfect price elasticity. An immediate consequence of these assumptions is that $\frac{dP_S}{dS_{CB}} = 0$. To simplify things further, we assume that banks, faced with higher deposits, will demand more long bonds to increase the duration of their portfolios. At the same time, we assume that they will not increase their demand for assets that are similar to the reserves they obtain as their deposits increase following the QE purchases, so $\frac{\partial f_B^S}{\partial F_B} = 0$. Now, we can focus on the response of the price for the long bond

$$\frac{dP_L}{dS_{CB}} = -\frac{\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_S} + P_L \frac{\partial f_{NB}^L}{\partial P_S})}{\Delta}, \tag{75}$$

where

$$\begin{aligned}
\Delta &= \alpha\delta - \beta\gamma \\
&= \left(\frac{\partial f_{NB}^L}{\partial P_S} + \frac{\partial f_B^L}{\partial P_S} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_S} + P_L \frac{\partial f_{NB}^L}{\partial P_S})\right)\left(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L}\right) \\
&\quad -\left(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_L} + P_L \frac{\partial f_{NB}^L}{\partial P_L})\right)\left(\frac{\partial f_{NB}^S}{\partial P_S} + \frac{\partial f_B^S}{\partial P_S}\right).
\end{aligned}$$

Next, we assume that the demand for short bonds by nonbank financial institutions is characterized by perfect price elasticity, i.e., $\frac{\partial f_{NB}^S}{\partial P_S} \to \infty$. We also assume that banks do not vary their demand for short bonds in response to changes in the short bond price, i.e., $\frac{\partial f_B^S}{\partial P_S} = 0$. This gives

$$\frac{dP_L}{dS_{CB}} = \frac{\frac{\partial f_B^L}{\partial F_B} P_S}{-\frac{\partial f_B^L}{\partial F_B} P_S(\frac{\partial f_{NB}^S}{\partial P_L} + \frac{\partial f_B^S}{\partial P_L}) - \left(\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - \frac{\partial f_B^L}{\partial F_B}(P_S \frac{\partial f_{NB}^S}{\partial P_L} + P_L \frac{\partial f_{NB}^L}{\partial P_L})\right)}. \tag{76}$$

Assume further that all cross-price elasticities are zero, i.e., we are in the extreme case of no substitution effects between assets. This reduces the expression to

$$\frac{dP_L}{dS_{CB}} = \frac{-\frac{\partial f_B^L}{\partial F_B} P_S}{\frac{\partial f_{NB}^L}{\partial P_L} + \frac{\partial f_B^L}{\partial P_L} - P_L \frac{\partial f_B^L}{\partial F_B}\frac{\partial f_{NB}^L}{\partial P_L}} > 0. \tag{77}$$

This shows that the price of long bonds can be positively affected when the central bank engages in quantitative easing by buying short-term bonds, as was the case in Switzerland in August 2011.

Finally, we note that fixing the cross-price elasticities to zero is not necessary to obtain this result. It remains valid as long as the cross-price elasticities are smaller in absolute value than the direct own-price elasticities.

The condition for positivity makes economic sense. It says that the rate at which banks increase their purchases of long-term bonds in response to an increase in deposit funding, times the value of those bonds, should not be larger than the increase in deposits itself.

Under our zero lower bound assumptions, with nonbanks being the marginal sellers of short bonds in a central bank QE program where only short bonds are bought, all short bonds are sold by nonbanks. These will, in return, hold more deposits with their correspondent bank. The price on short bonds will only increase infinitesimally to make this shift happen, as we are at the zero lower bound. If the correspondent banks do not react to the increase in their deposits, there would be no further price changes. However, we assume that banks do react, by increasing their demand for long bonds in an effort to increase the duration of their portfolios. This generates upward pressure on long-term bond prices, or a drop in their term premiums.

If, on the other hand, banks are the sellers of short-term bonds to the central bank under a QE program at the zero lower bound, then there is no effect on asset prices. Banks effectively swap short bonds for reserves, being indifferent between the two, and see no changes to their duration as a result.

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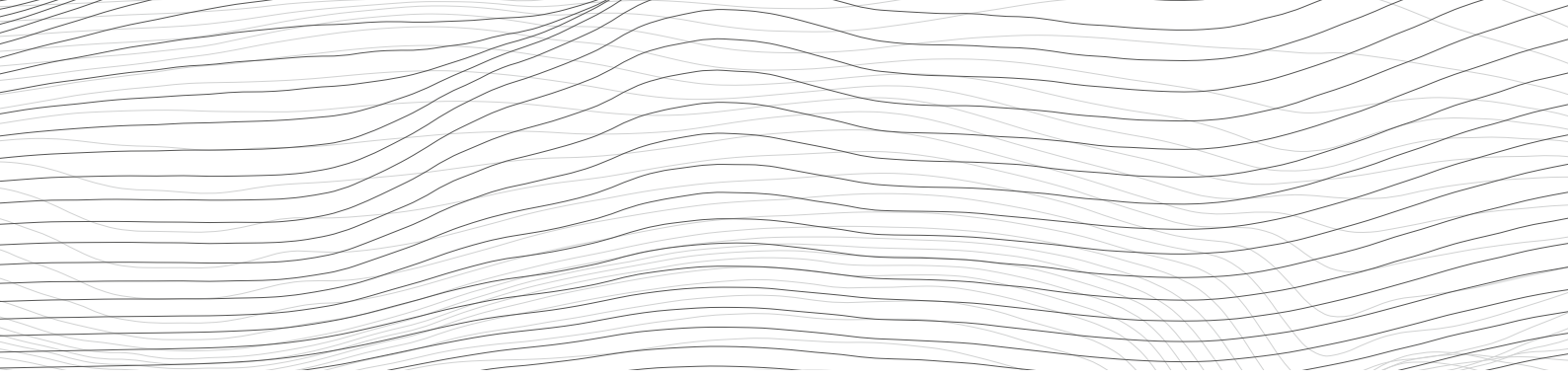
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