Macroeconomic and interest rate volatility under alternative monetary operating procedures

Petra Gerlach-Kristen and Barbara Rudolf

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Petra Gerlach-Kristen
Bank for International Settlements

and

Barbara Rudolf[*]
Swiss National Bank

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Abstract

During the financial crisis of 2007/08 the level and volatility of interest rate spreads increased dramatically. This paper examines how the choice of the target interest rate for monetary policy affects the volatility of inflation, the output gap and the yield curve. We consider three monetary policy operating procedures with different target interest rates: two market rates with maturities of one and three months, respectively, and an essentially riskless one-month repo rate. The implementation tool is the one-month repo rate for all three operating procedures. In a highly stylised model, we find that using a money market rate as a target rate generally yields lower variability of the macroeconomic variables. This holds under discretion as well as under commitment both in times of financial calm or turmoil. Whether the one month or three month rate procedure performs best depends on the maturity of the specific rate that enters the IS curve.

1 Introduction

Monetary policy operating procedures vary from one country to another. In particular, there are major differences regarding the target interest rate used to formulate monetary policy. Some central banks have as operational target a short-term market interest rate while others use the rate they charge financial institutions for the provision of short-term funds – typically a repo rate. Since short-term interest rates in a given currency tend to move closely in line with each other during normal times, these differences have attracted little attention in the literature.[1] In the financial crisis of 2007/08, however, the level and volatility of interest rate spreads increased dramatically, raising the issue of how alternative monetary policy procedures impact on the economy. In this paper, we examine how the choice of monetary operating procedure affects the volatilities of the inflation rate, the output gap and the term structure of market rates.

The model is standard in that it consists of a hybrid New Keynesian Phillips curve and a consumption Euler equation. The novelty is the consideration of three possible interest rates for monetary policy: a repo rate with a one-period maturity and two money market rates with maturities of one and three periods, respectively. Policy may be formulated with either of these rates as target rate, but is always implemented with the one-period repo rate. The market rates are modelled to depend on the expected future path of the repo rate, a term premium and a risk premium. Since risk premia are largely driven by markets' perception of default risk, we assume that these premia are linked to economic conditions and let them depend endogenously on the expected future path of the output gap. Following the literature, we let the one-month money market rate impact on the output gap in the IS curve in the baseline model, but we also consider a variant of the model where the average of short and longer-term money market rates matters.

We compare the three monetary operating procedures by examining optimal policy reaction functions, impulse responses and simulated volatilities of inflation, the output gap and the yield curve. This is done first for a baseline calibration, using parameters chosen to mirror pre-crisis conditions, and then for an alternative set of parameters which reflects the financial turmoil. Results are presented for both, policy under commitment and under discretion. We then consider how the results change if an average interest rate that is constructed from market rates of maturities lasting from one to twelve months matters for economic activity. Finally,

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[1]Exceptions are Bindseil [5] and Borio [6]. Borio and Nelson [7] discuss monetary operations during the financial crisis.

since there have been calls for using monetary policy to stabilise asset prices, we examine how an additional target for stabilising the yield curve impacts on volatilities under the three operating procedures.

The results suggest that under commitment, the three operating procedures give similar results in terms of macroeconomic volatility in tranquil times. Under discretion, market-based procedures yield a more stable macroeconomy. In times of financial turmoil, targeting the short-term market interest rate is advantageous both under commitment and discretion. If we depart from the baseline model and assume that also longer-term market rates matter for the output gap, the different procedures yield equal volatility if policy is set under commitment. Under discretion, the approach where monetary policy is formulated it terms of a longer-term market rate seem most promising. Finally, if the central bank tries to stabilise the yield curve, for instance because too low a level of the term structure can give rise to asset price booms, our model shows a feedback effect that increases not only the volatility of inflation and the output gap, but also of the yield curve itself.

This paper adds to a growing literature that models the linkages between financial market interest rates, risk premia and the macroeconomy. Goodfriend and McCallum [20] assume an interbank policy interest rate, a risk-free rate, and collateralised and uncollateralised market rates in an economy with a banking sector and discuss the responses of these interest rates to shocks. Cúrdia and Woodford [9] model the spread between borrowing and lending rates and show that monetary policy provides better results if the central bank reacts to movements in the credit spread. The reason is that the rate relevant for economic activity is not the policy rate itself but an interest rate that depends on the credit spread.[2] Eijffinger, Schaling and Verhagen [12], Fendel [17], Lansing and Trehan [27] and Svensson [37] present optimal policy rules for a short-term interest rate in models where a longer-term rate, which obeys the expectations hypothesis, matters for the output gap. Conversely, Kulish [26] and McGough, Rudebusch and Williams [30] let the shorter-term market rate enter the IS curve and analyse different reaction functions for the longer-term interest rate, which, however are not derived optimally. Gerlach-Kristen and Rudolf [19], finally, compare the performance of Taylor rules for both a short and a longer-term interest rate when the latter matters for economic activity.

The paper is organised as follows. Section 2 gives a brief description of policy rates and

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[2]Martin and Milas [29] examine the spread between the monetary policy rate and an economically relevant borrowing rate and discuss how monetary policy in the UK responded to movements in market rates.

market interest rates in the US, the UK and Switzerland over the period 2005-2008. Section 3 introduces the model. Section 4 discusses optimal policy for the three operating procedures and the resulting volatilities for inflation, the output gap and the yield curve. Section 5 concludes.

2   Interest rates during the 2007/08 financial crisis

The analysis in this paper is loosely modelled on the monetary policy operating procedures of three central banks: the Bank of England, the Federal Reserve and the Swiss National Bank.[3] The operating procedures of these central banks differ little with respect to the implementation tool: all use repo rates with very similar short-term maturities.[4] However, the three banks express their monetary policy intentions in terms of a target for interest rates which differ quite substantially.

The Bank of England formulates monetary policy in terms of Bank rate, i.e. the repo rate at which the Bank is willing, against eligible collateral, to lend funds to commercial banks. The typical maturity of these repo transactions, which are essentially risk free, is one week. The Federal Reserve's operational target is the federal funds rate, i.e. the rate at which commercial banks lend uncollateralised overnight funds to one another. Thus, the US target rate is a market rate at the very short end of the maturity spectrum which incorporates default and other risks. Before the financial crisis, the Federal Reserve influenced the level of the federal funds rate through repo transactions with overnight and two-week maturity. The Swiss National Bank, finally, announces monetary policy in terms of a target range of typically one percentage point for the three-month CHF libor, which is a rate for uncollateralised three-month funds on the London interbank market. The Swiss National Bank implements its policy using repos of typically one-week maturity.[5]

To illustrate the effect of monetary policy operating procedures on interest rates, Figure

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[3]The European Central Bank is not considered in this study since the policy rate is the minimum bid rate, which before October 2008 often deviated from the actual price paid by the commercial banks for central bank funds in the variable rate tenders (after the collapse of Lehman Brothers, the ECB adopted fixed-rate tenders). Modelling such a framework is beyond the scope of this paper.

[4]The Federal Reserve moreover buys and sells securities as part of its open market operations. In the crisis, additional instruments such as the Term Auction Facility were introduced.

[5]In a similar setup, the Bank of Canada had a target for the three-month Treasury bill rate until January 1996 (Borio [6]). Regarding the implementation of monetary policy in Switzerland, see Jordan and Kugler [23] and Jordan, Ranaldo and Söderlind [24].

Figure 1: Interest rates, January 2005 to January 2009
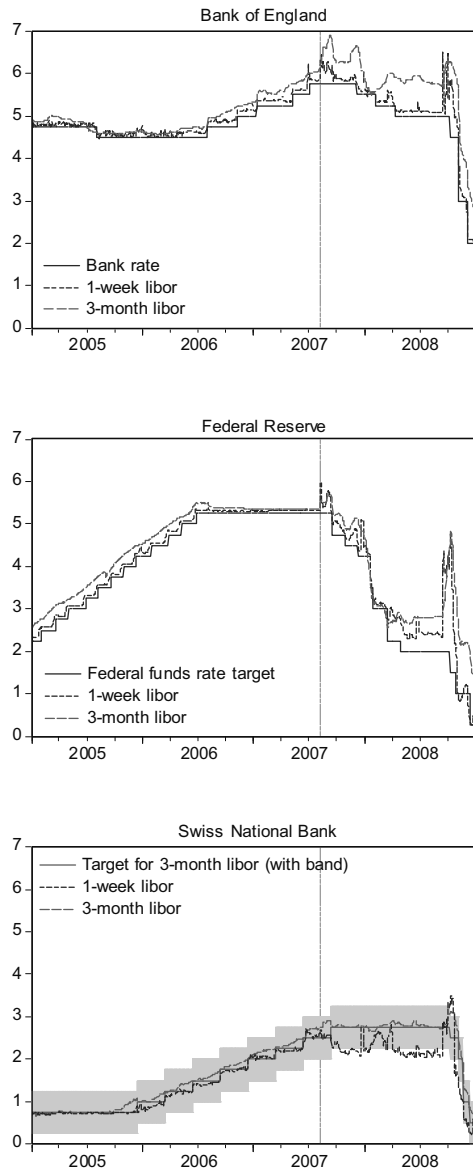
Note: The vertical dashed line represents the onset of the crisis in August 2007.

1 shows interest rates for the US, the UK and Switzerland over the period January 2005 to January 2009. Interest rates moved closely together before the onset of the crisis in August 2007. The volatility of the spreads then increased and peaked in September and October 2008 after the collapse of Lehman Brothers. It is notable that the three-month libors were more volatile than the one-week libors for GBP and USD, but that the opposite is true for CHF. This is because the Swiss National Bank stabilised the three-month libor by adjusting its short-term repo rates. These repo rate adjustments were translated into movements at the very short end of the yield curve, which consequently was more volatile for CHF than for GBP and USD. The choice of target rate for monetary policy appears to determine which part of the yield curve is stabilised and which maturities adjust to risk shocks, a phenomenon Flemming [18] referred to as "pivoting". This choice in turn might impact on the volatility of the macroeconomy.

3 The model

To study how the choice of monetary operating procedure impacts on macroeconomic volatility, we consider an extended version of the canonical New Keynesian model (see e.g. Woodford [42]). We first describe the economy and then discuss the monetary policy problem, which consists of deriving the optimal reaction function for the repo rate, which is the central bank's implementation tool.

3.1 The economy

The model economy consists of a hybrid New Keynesian Phillips curve, a consumption Euler equation and a set of equations describing the dynamics of the term structure of interest rates. The hybrid New Keynesian Phillips curve is given by

$$\pi_t = a_\pi E_t \pi_{t+1} + (1 - a_\pi)\pi_{t-1} + a_y y_t + u_{\pi,t}, \tag{1}$$

where π_t is the inflation rate, y_t the output gap, a_π a parameter reflecting the degree of forward-lookingness in the price-setting behaviour of firms, and a_y a composite parameter capturing the discount rate and the frequency of price adjustments. The exogenous inflationary shock, $u_{\pi,t}$, is assumed to follow an AR(1) process,

$$u_{\pi,t} = \rho_\pi u_{\pi,t-1} + \sigma_\pi e_{\pi,t} \tag{2}$$

with $0 < \rho_\pi < 1$ and $e_{\pi,t} \sim N(0,1)$.

The log-linearised consumption Euler equation is given by

$$y_t = b_y E_t y_{t+1} + (1 - b_y) y_{t-1} - b_r (i_{1,t} - E_t \pi_{t+1} - \mu_{1,r}) + u_{y,t}, \tag{3}$$

where $i_{1,t}$ denotes the annualised nominal money market interest rate with maturity of one period, $\mu_{1,r}$ the equilibrium one-period real market rate, and $u_{y,t}$ an exogenous demand shock which evolves according to

$$u_{y,t} = \rho_y u_{y,t-1} + \sigma_y e_{y,t} \tag{4}$$

with $0 < \rho_y < 1$ and $e_{y,t} \sim N(0,1)$. Thus, we assume that economic activity depends on the short-term market rate. This assumption is common in the literature. Empirical arguments why economic activity depends mainly on short-term rates include the fact that banks funds themselves mostly at that horizon and that mortgages often are priced off short-term money market rates. That said, policymakers often argue that long-term rather than short-term interest rates impact on economic activity (see e.g. Bernanke [4] and Rudebusch, Sack and Swanson [34]). We therefore consider as a robustness test in Section 4.3 an alternative IS curve in which the average real interest rate matters. In particular,

$$y_t = b_y E_t y_{t+1} + (1 - b_y) y_{t-1} - b_r (i_{av,t} - E_t \pi_{av,t+1} - \mu_{av,r}) + u_{y,t}, \tag{5}$$

where $i_{av,t}$ is the average of the money market rates $i_{j,t}$ with maturities $j = 1$ to 12 months,

$$i_{av,t} = \frac{1}{12} \sum_{j=1}^{12} i_{j,t}, \tag{6}$$

and average inflation is defined accordingly,

$$\pi_{av,t+1} = \frac{1}{12} \sum_{j=1}^{12} E_t \pi_{j,t+1}, \tag{7}$$

with $\pi_{j,t}$ the annualised inflation rate over the next j months. Since we model interest rate only out to a horizon of twelve months, this alternative IS curve should be seen as an illustration rather than a realistic description of the role of longer-term rates in the economy. Nevertheless, as Section 4.3 below shows, even within these limitations there are palpable changes in the results.[6]

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[6] We do not consider an IS curve with only a longer-term rate entering since this raises indeterminacy problems in the 3MR procedure, which we discuss in Section 3.2 below. The reason for this is that if the one-month rate does not enter the IS, it is not pinned down anymore (in the RR and the 1MR procedures, the smoothing objective discussed below achieves this).

The novel part of the model is that the central bank does not directly control the market rate entering the IS curve, but the repo rate i_t. The short-term money market rate $i_{1,t}$ deviates from the repo rate by a risk premium $\theta_{1,t}$, so that

$$i_{1,t} = i_t + \theta_{1,t}. \tag{8}$$

This risk premium reflects counterparty risk that arises in a market transaction but does not figure in transactions with the central bank.[7] We concentrate on default risk, which we see as being dependent on the state of the business cycle. In particular, we assume that the default risk on a loan increases when the output gap is expected to fall.[8] Thus, the expected output gap at the end of the credit contract matters for the risk premium of the one-period money market interest rate, and we write

$$\theta_{1,t} = \theta_1 - c E_t y_{t+1} + \varepsilon_{1,t}, \tag{9}$$

where θ_1 captures the constant component of the default risk and c denotes the impact of economic activity. The innovation in equation (9) follows an AR(1) process,

$$\varepsilon_{1,t} = \rho_\varepsilon \varepsilon_{1,t-1} + \sigma_{e,1} e_{1,t},$$

with $e_{1,t} \sim N(0,1)$. Longer-term money market interest rates are given by the expectations hypothesis. The j-period interest rate $i_{j,t}$ is defined as

$$i_{j,t} = \tau_j + \theta_{j,t} + \frac{1}{j} E_t \sum_{k=0}^{j-1} i_{t+k}, \tag{10}$$

where τ_j denotes a constant term premium and $\theta_{j,t}$ the j-period risk premium.[9] The latter, in

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[7]Michaud and Upper [31] offer a detailed discussion of the evolution of risk premia during the financial crisis. Using daily data, they find that liquidity matters at high frequencies, while default risk appears to impact at lower frequencies.

[8]Fama and French [16] show that default spreads between risky and essentially riskless bonds are high if business conditions are weak, and Campbell, Lo and MacKinlay [8] argue that risk aversion is time-varying either because of habit formation or agents' heterogeneity. Affine models of the term structure document the empirical relationship between the yield curve and the state of the business cycle as well as inflation (see Ang and Piazzesi [2], Dewachter and Lyrio [11], Hördahl, Tristani and Vestin [22], Kozicki and Tinsley [25] and Piazzesi and Swanson [32]). Emiris [14] studies the term structure of interest rates in a DSGE model and finds that premia are related to shocks to consumption and investment, thus providing a theoretical link between market interest rates and the business cycle.

[9]As an alternative to the expectations hypothesis, Amisano and Tristani [1], Atkeson and Kehoe [3] and Emiris [14] derive expressions for longer-term interest rates in micro-based models with frictions.

turn, depends on the expected future path of the output gap,

$$\theta_{j,t} = \theta_j - c\frac{1}{j}E_t \sum_{k=1}^{j} y_{t+k} + \varepsilon_{j,t}, \tag{11}$$

with

$$\varepsilon_{j,t} = \rho_\varepsilon \varepsilon_{j,t-1} + \sigma_{e,j} e_{j,t}, \tag{12}$$

where the risk innovations $e_{j,t} \sim N(0,1)$ are correlated across maturities.[10]

Finally, we assume that the central bank and the private sector form rational expectations and have access to the same information about the economy.

3.2 The monetary policy problem

We assume that the central bank's period loss function is given by

$$L_t = \frac{1}{2}Y_t'\Lambda Y_t, \tag{13}$$

and the intertemporal loss function by

$$\pounds_0 = E_0 \sum_{t=0}^{\infty} (1-\delta)\delta^t L_t, \tag{14}$$

where Y_t is a vector of goal variables, δ the discount factor and Λ the matrix of goal weights that differ between operating procedures. Under the first of our three monetary operating procedures, policy is formulated with the repo rate i_t. Thus, the target rate of monetary policy and the implementation tool coincide. We refer to this approach as the repo rate operating procedure (RR procedure). Alternatively, policy can seek to steer as target rate the one-period money market rate $i_{1,t}$. This is labeled the one-month money market rate (1MR) procedure. Under the three-month money market rate (3MR) procedure, finally, policy targets the three-period money market rate $i_{3,t}$.

The central bank minimises variations in inflation and in the output gap under all three operating procedures. Moreover, the central bank is assumed to smooth the target rate, i.e. i_t, $i_{1,t}$ or $i_{3,t}$, depending on the procedure. This assumption is supported by the observation that monetary policy tends to be changed gradually with no obvious attempts being made to smooth movements of interest rates at other maturities.[11] The set of all potential goal variables

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[10]Svensson [36] discusses the links between different types of premia across different maturities.

[11]On interest rate smoothing, see e.g. Ellis and Lowe [13], English, Nelson and Sack [15], Goodhart [21] and Rudebusch [33].

in equation (13) then is

$$Y_t = \begin{bmatrix} \pi_t & y_t & \Delta i_t & \Delta i_{1,t} & \Delta i_{3,t} \end{bmatrix}',$$

where the off-diagonal elements of Λ are zero for all operating procedures. The diagonal is given by $\begin{bmatrix} \lambda_\pi & \lambda_y & \lambda_i & 0 & 0 \end{bmatrix}$ under the RR procedure, by $\begin{bmatrix} \lambda_\pi & \lambda_y & 0 & \lambda_i & 0 \end{bmatrix}$ under the 1MR procedure and by $\begin{bmatrix} \lambda_\pi & \lambda_y & 0 & 0 & \lambda_i \end{bmatrix}$ under the 3MR procedure, where λ_π is the weight attached to the goal of stabilising inflation, λ_y the weight attached to output gap stabilisation and λ_i the weight attached to target rate smoothing. Thus, the only difference between procedures is the specific target rate smoothed.

To determine the optimal reaction function for the central bank's repo rate, we minimise the loss function with respect to the repo rate i_t, subject to the structure of the economy as given by equations (1) to (12). The appendix discusses the state space representation of the model and the optimisation under commitment and discretion in detail.

Before proceeding further, a technical difficulty in computing optimal policies for the market rate-based procedures (1MR/3MR) should be noted. Under these procedures, policymakers smooth either $i_{1,t}$ or $i_{3,t}$, which includes a risk premium that is a function of the expected future path of the output gap.[12] That path, in turn, depends on monetary policy and thus on the current repo rate i_t, which is the variable the optimisation is solved for. In other words, defining the loss function for the 1MR/3MR procedure presupposes the knowledge of the optimal reaction function for i_t that minimises this loss function. We tackle the problem by guessing an initial reaction function and iterating until convergence.

4   Results

In this section, we first compare the three operating procedures in a baseline specification, assuming that monetary policy is conducted either under commitment and in a timeless perspective (see Svensson and Woodford [39] and Woodford [42]) or under discretion (see Söderlind [35]). We compute the optimal repo rate reaction functions, the impulse response functions and the average volatilities of inflation, the output gap and the yield curve. We then discuss how the results change during financial turmoil, i.e. when risk shocks are larger and more correlated. Next, we consider the variant of the model with the average of the money market rates of hori-

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[12]We assume that the expected output gap depends in the same way from expected other variables as the current output gap depends on their current values.

zons one to twelve months in the IS curve. Finally, we study how stabilising the level of the yield curve impacts on the results.

4.1 Baseline case

The periodicity of the model is assumed to be monthly. The baseline calibration sets the coefficients in the Phillips curve, the consumption Euler equation and the corresponding shock processes to $a_\pi = b_y = 0.8$, $a_y = 0.2$, $b_r = 0.5$, $\rho_\pi = \rho_y = 0.9$, $\rho_\varepsilon = 0.5$ and $\sigma_\pi = \sigma_y = 0.1$.[13] Central bank preferences are specified by the weights put in the period loss function on inflation stabilisation, output gap stabilisation and target rate smoothing, which we set to $\lambda_\pi = \lambda_y = 1$ and $\lambda_i = 0.5$, and by the discount factor $\delta = 0.999$. For the risk premia, we set $\theta_1 = \theta_j = 0.1$, $c = 0.25$, $\sigma_{e,j} = 0.01$ and $\sigma_{e,jk} = 0.001$ for $j \neq k$. Finally, the constant term premia are modelled as $\tau_j = \sqrt{j-1}/10$, a functional form that matches US pre-crisis data rather well. Given these parameters, we minimise the loss function and obtain the optimal reaction function for the repo rate.

Optimal repo rate reaction functions: Table 1 shows the optimal repo rules for the three operating procedures for commitment (upper panel) and discretion (lower panel). Under commitment, the RR procedure calls for essentially no response of the repo rate to past inflation (-0.005) and the past output gap (0.084), but a stronger reaction to innovations in the Phillips curve (0.388) and in the IS curve (0.983). There is considerable repo rate smoothing (0.463) and a negative response of the repo rate to shocks in the one-month risk premium (-0.339), implying that monetary policy partially absorbs such shocks. Thus, if the risk premium rises, the repo rate is lowered so that the increase in the one-month market rate is smaller than that in $\theta_{1,t}$. However, the repo rate does not fully absorb the shock, since this would require a sharp response that conflicts with the goal of smoothing the target rate. Finally, shocks to the three-month risk premium trigger no repo rate reaction, and the responses to the Lagrange multipliers for the Phillips and IS curves, ξ_{t-1}^{PC} and ξ_{t-1}^{IS}, which capture the importance of future interest rate setting under commitment, are small (-0.017 and 0.150).

The 1MR procedure yields a similar optimal repo rate reaction function. The main difference is that innovations in the one-month risk premium are fully absorbed. This implies that risk

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[13] Robustness tests show that the results presented here do not depend qualitatively on the exact parameter assumptions. It should be noted that, because of the monthly periodicity, the AR coefficients chosen are larger than in the standard literature, which assumes quarterly periodicity, and the standard errors are smaller.

Table 1: Optimal reaction functions in the baseline case

					Commitment						
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{3,t}$	ξ^{PC}_{t-1}	ξ^{IS}_{t-1}
RR	-0.005	0.084	0.463	0	0	0.388	0.983	-0.339	0	-0.017	0.150
1MR	-0.013	0.074	0	0.453	0	0.266	1.003	-1	0	-0.026	0.115
3MR	-0.000	0.163	0	0	0.651	0.164	0.699	-0.365	-1.089	-0.013	0.385

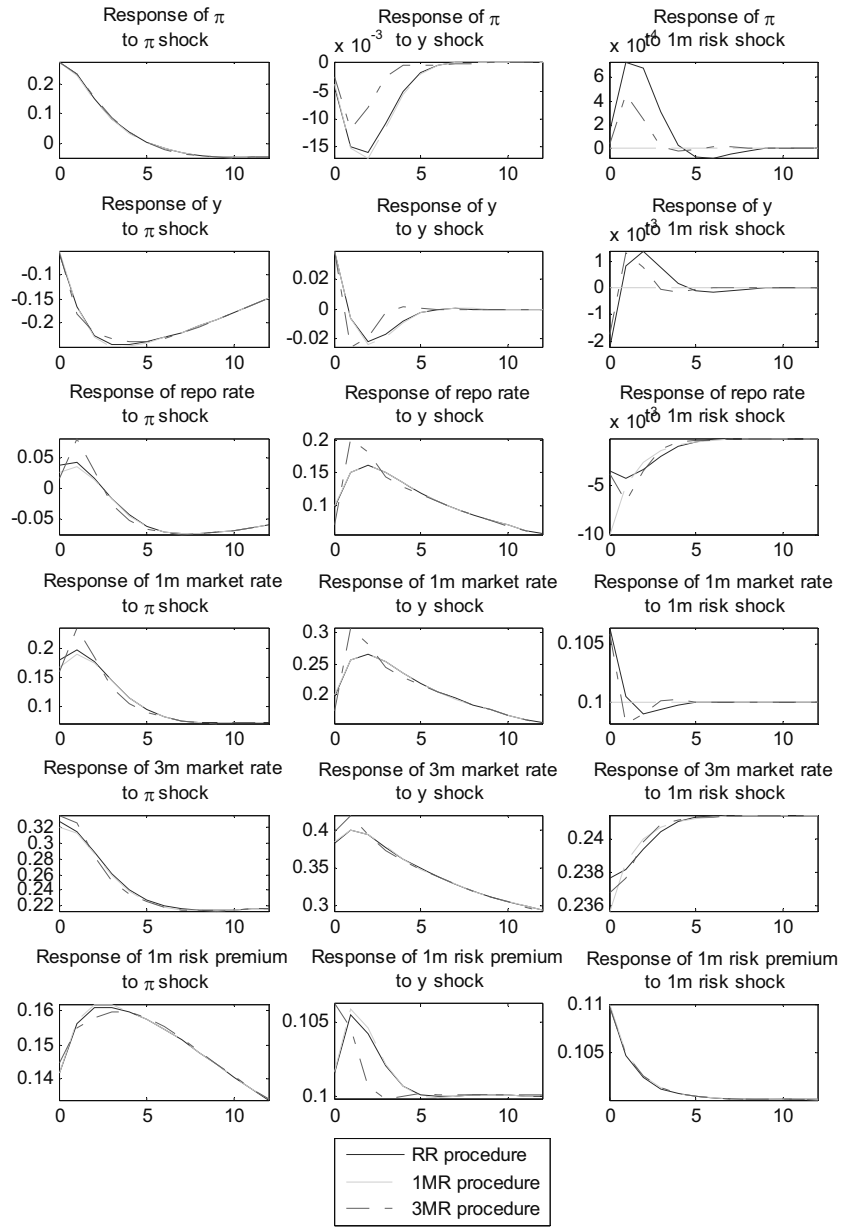|					Discretion				
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{3,t}$
RR	0.062	0.131	0.259	0	0	4.124	1.417	-0.528	0
1MR	0.060	0.129	0	0.252	0	3.790	1.432	-1	0
3MR	0.105	0.263	0	0	0.315	3.966	1.287	-0.708	-0.582

Note: Repo rate reaction function coefficients for different operating procedures. RR, 1MR and 3MR stand for repo rate, one-month and three-month money market rate procedures.

shocks at the one-month horizon do not change the one-month money market rate and thus have no impact on the output gap and inflation. Full absorption is possible under the 1MR procedure since the short-term money market rate is smoothed, rather than the repo rate.

The 3MR procedure calls for a weaker response to inflation and output gap shocks. Innovations in the one-month risk premium are partly, and three-month shocks essentially fully, absorbed. Finally, there is considerable interest rate smoothing with respect to the three-month market rate.

Under discretion, the coefficients on inflation and output gap shocks are much larger than under commitment, and interest rate smoothing is less pronounced. These changes reflect the stabilisation bias discussed in Dennis and Söderström [10] and Woodford [41]. In an economy with forward-looking agents, interest rate smoothing under commitment stabilises expectations and thereby reduces overall macroeconomic volatility. If policy is discretionary, however, policy-makers do not follow this optimal gradual response and therefore stabilise the output gap more and inflation less. Compared with commitment, the RR and the 3MR procedures react more to one-month risk premium shocks. Three-month shocks trigger a smaller response under the 3MR procedure, again because there is less weight attached to variables relating to the future.

Figure 2: Impulse response functions under commitment

Note: RR, 1MR and 3MR stand for repo rate, one-month
and three-month money market rate procedures.

Figure 3: Impulse response functions under discretion
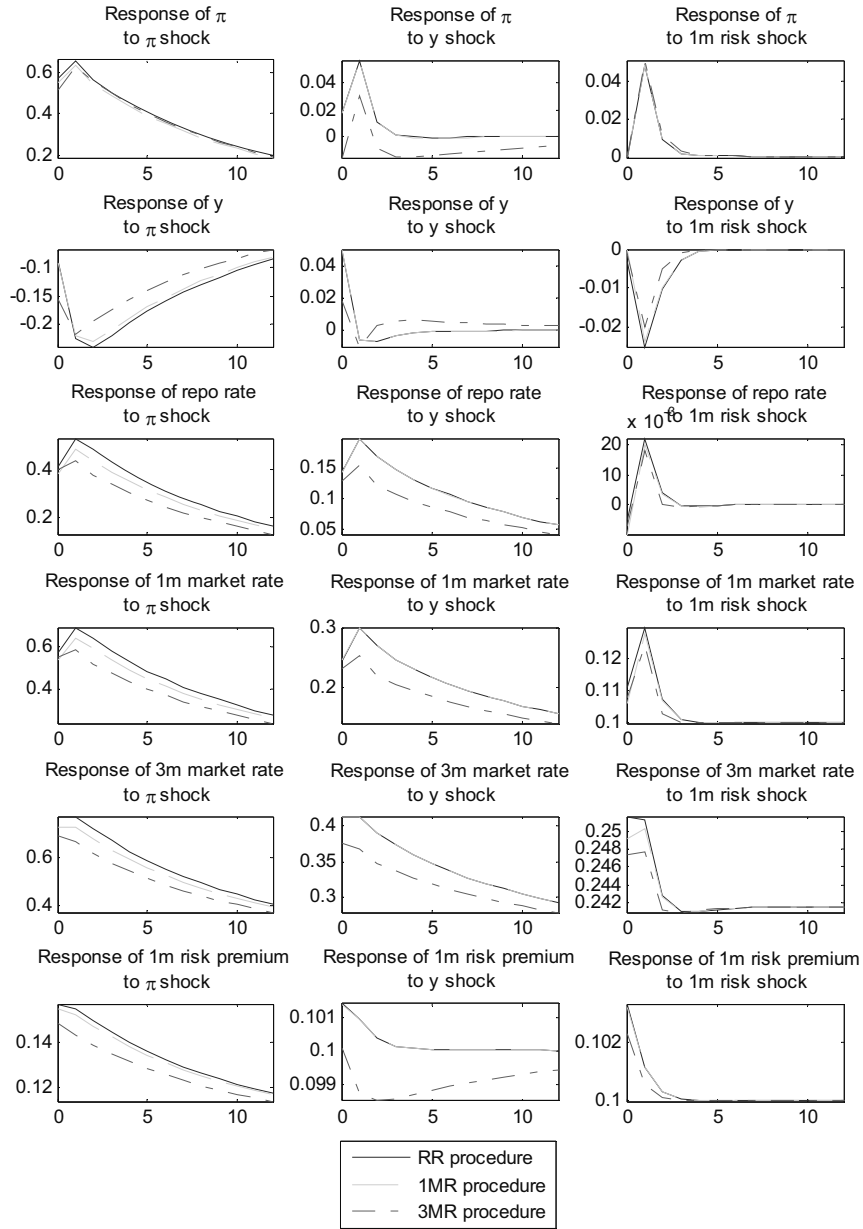
Note: RR, 1MR and 3MR stand for repo rate, one-month
and three-month money market rate procedures.

Impulse responses: To evaluate how the choice of monetary operating procedure affects macroeconomic dynamics, we present impulse responses for various shocks under the RR, 1MR and 3MR procedures. Figure 2 shows the responses under commitment, Figure 3 those under discretion. The first two columns illustrate the impact of a one-standard-deviation shock to inflation and the output gap, respectively, and the last column shows the effect of a shock to the one-month risk premium.

For the case of commitment, the first two plots in the last column of Figure 2 indicate that a risk shock affects inflation and the output gap most under the RR procedure, followed by the 3MR approach. The reason for this is that the cut in the repo rate in response to a risk shock is under these procedures not sufficient to absorb the shock completely. The rise in the risk premium thus implies an increase in the one-month market rate. By contrast, absorption is complete under the 1MR procedure and as a result, the one-month market rate, inflation and the output gap are essentially unaffected. The three-month market rate declines strongly on impact because of the reduction in the repo rate.

The impulse responses to an inflationary shock are almost identical for all three procedures, reflecting the broad similarities between their optimal reaction functions. A positive inflation shock is undone slowly by tighter monetary policy, which causes the output gap to turn negative. Because of the fall in output, default risk increases and the market rates rise, thus further depressing output before the variables return to equilibrium. A positive output gap shock, finally, drives the repo rate up, and inflation and subsequently the output gap turn negative for some time. Under the 3MR procedure, the repo rate is raised most strongly, so that the responses of inflation and the output gap to the shock are smaller and faster. The market rates rise in response to an output gap shock under all operating procedures due to the tightening of the repo rate.

For the case of discretion, Figure 3 shows similar responses to an inflation shock. Output gap shocks trigger a more aggressive initial repo rate response because of the stabilisation bias and a faster subsequent reduction of that rate. This makes the output gap return to equilibrium quickly but leads to a temporary build-up in inflation. Similarly, the output gap is stabilised fast after a risk premium shock, at the expense of temporarily higher inflation.

Macroeconomic and interest rate volatility: To compare the monetary operating procedures in terms of welfare we next compute the volatility of the inflation rate, the output gap

Figure 4: Volatilities in the baseline case
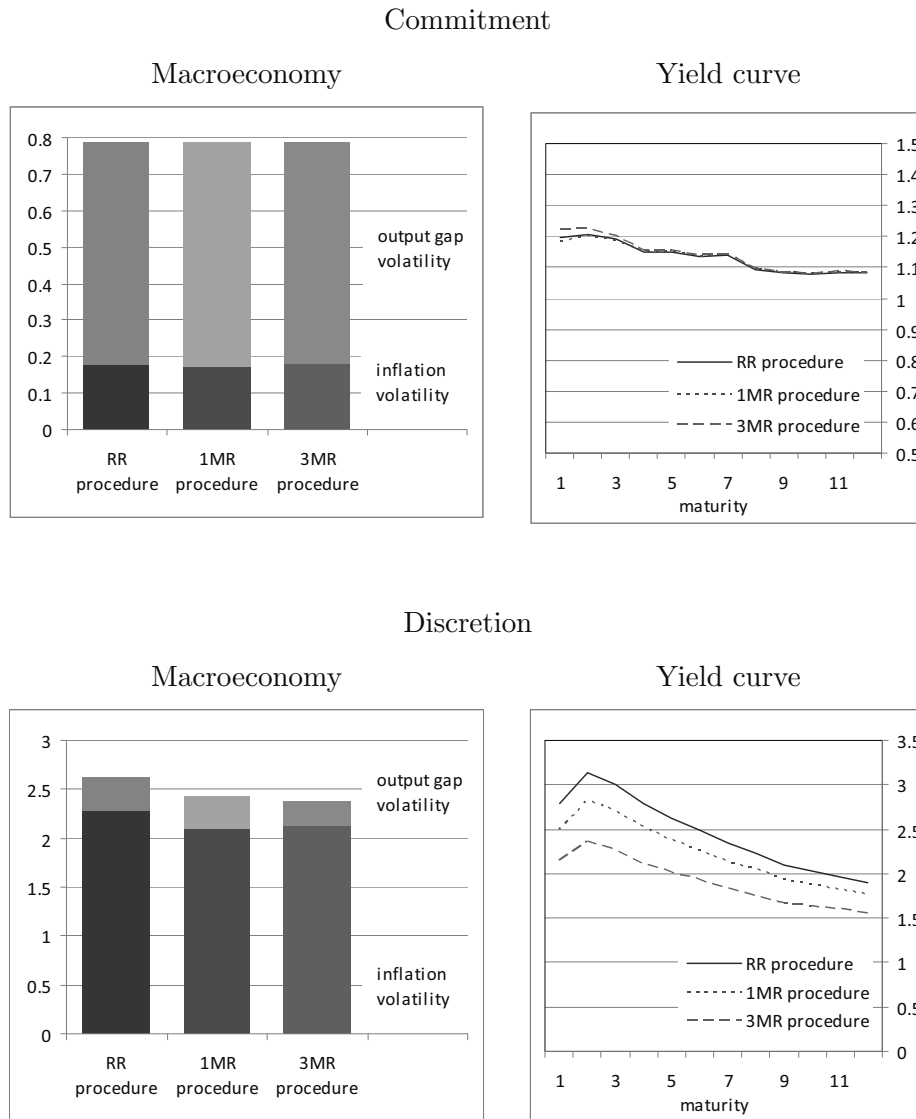
Commitment

Macroeconomy

Yield curve

Discretion

Macroeconomy

Yield curve

Note: Simulations with 10,000 draws. RR, 1MR and 3MR stand
for repo rate, one-month and three-month money market rate procedures.

and the yield curve. For that purpose we perform stochastic simulations. We generate $10,000$ draws for each monetary operating procedure. The upper panel of Figure 4 presents the results for commitment, the lower panel for discretion. The left plots show the simulated volatilities for the macroeconomic variables, while volatilities of the market interest rates with maturity $j = 1, ..., 12$ months are shown in the right plots. In interpreting these volatilities, it is important to remember that the results are biased in favour of the short-term money market rate. This bias arises because of the assumption that this rate matters in the IS curve.

Under commitment, the variability of the macroeconomic variables and the yield curve are virtually identical for the three monetary operating procedures. Under discretion, macroeconomic volatility is higher because monetary policy does not attempt to impact on expectations. Also, the stabilisation bias for the output gap relative to inflation is clearly visible. Macroeconomic volatility is lowest under the 3MR and 1MR procedures. It seems thus that using a policy target that refers to a market rate is preferable to a repo rate target. The yield curve is more volatile than under commitment, reflecting the stronger response of policy to shocks. The variability at the short end of the term structure is comparatively low because it is optimal to limit the volatility of the one-month market rate since it enters the IS curve.

Overall, the baseline results suggest that it does not matter much which monetary operating procedure is chosen in times of financial calm if policymakers can commit themselves. This might explain why different approaches are observed in practice. If policy is set in a discretionary fashion, volatilities are lower if a money market rate, rather than the repo rate, is chosen as target rate. We next ask how these results change in times of heightened financial risk.

4.2 Financial turmoil

In this section, we study how an increase in the variance and the correlation of the risk shocks similar to that in the financial crisis of 2007/08 affect the results. In particular, we assume an autocorrelation of the risk premium shocks of $\rho_\varepsilon = 0.9$ (instead of $\rho_\varepsilon = 0.5$) and multiply the standard errors by one hundred.

Arguably, it is more realistic to assume that monetary policy is set in a discretionary manner in times of crisis. For completeness, we again present also results assuming commitment.

Optimal repo rate reaction functions: Table 2 shows the optimal reaction functions for the three operating procedures. Both under commitment and discretion, the responses to the

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risk premia change for the RR and the 3MR procedure, but not for the 1MR approach. The reason for this is that we only assume different dynamics for the risk premium, movements of which the 1MR procedure absorbs fully. For the RR procedure, we now find a stronger response to changes in $\varepsilon_{1,t}$ because innovations in the one-month market rate are more protracted and therefore have a larger effect on inflation and the output gap. For the 3MR procedure, by contrast, we find a weaker response both to changes in $\varepsilon_{1,t}$ and $\varepsilon_{3,t}$. This result is due to the fact that the higher autocorrelation of the risk shocks generates the expectation that policy responds for several periods in the same direction after an initial innovation. This expectation causes ceteris paribus a large movement in longer-term market rates, which policymakers under the 3MR procedure however would like to smooth. Therefore, a smaller response to risk shocks becomes desirable as ρ_ε rises.

Table 2: Optimal reaction functions during financial turmoil

| | | | | | | Commitment | | | | | |
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{3,t}$	ξ_{t-1}^{PC}	ξ_{t-1}^{IS}
RR	-0.005	0.084	0.463	0	0	0.388	0.983	-0.491	0	-0.017	0.150
1MR	-0.013	0.074	0	0.453	0	0.266	1.003	-1	0	-0.026	0.115
3MR	-0.000	0.163	0	0	0.651	0.164	0.669	-0.350	-0.720	-0.013	0.385
						Discretion					
	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{3,t}$		
RR	0.062	0.131	0.259	0	0	4.124	1.417	-0.708	0		
1MR	0.060	0.129	0	0.252	0	3.790	1.432	-1.000	0		
3MR	0.105	0.263	0	0	0.315	3.966	1.287	-0.643	-0.351		

Note: Repo rate reaction function coefficients for different operating procedures. RR, 1MR and 3MR stand for repo rate, one-month and three-month money market rate procedures.

Macroeconomic and interest rate volatility: The volatilities under commitment are displayed in the upper panel in Figure 5. Comparing them with those from the baseline calibration of the model, we find that macroeconomic volatility has increased under the RR and 3MR procedures but remains essentially unchanged under the 1MR approach. This is again due to the fact that risk shocks are fully undone by the 1MR procedure. Macroeconomic volatility is

largest under the 3MR approach since this procedure absorbs risk premium shocks least. The yield curve volatility in the upper right panel is higher for all procedures due to the larger risk shocks at all horizons. For the 3MR procedure, the aim of smoothing the three-month rate is now visible as a kink in the yield curve volatility at that horizon.

Figure 5: Volatilities during financial turmoil
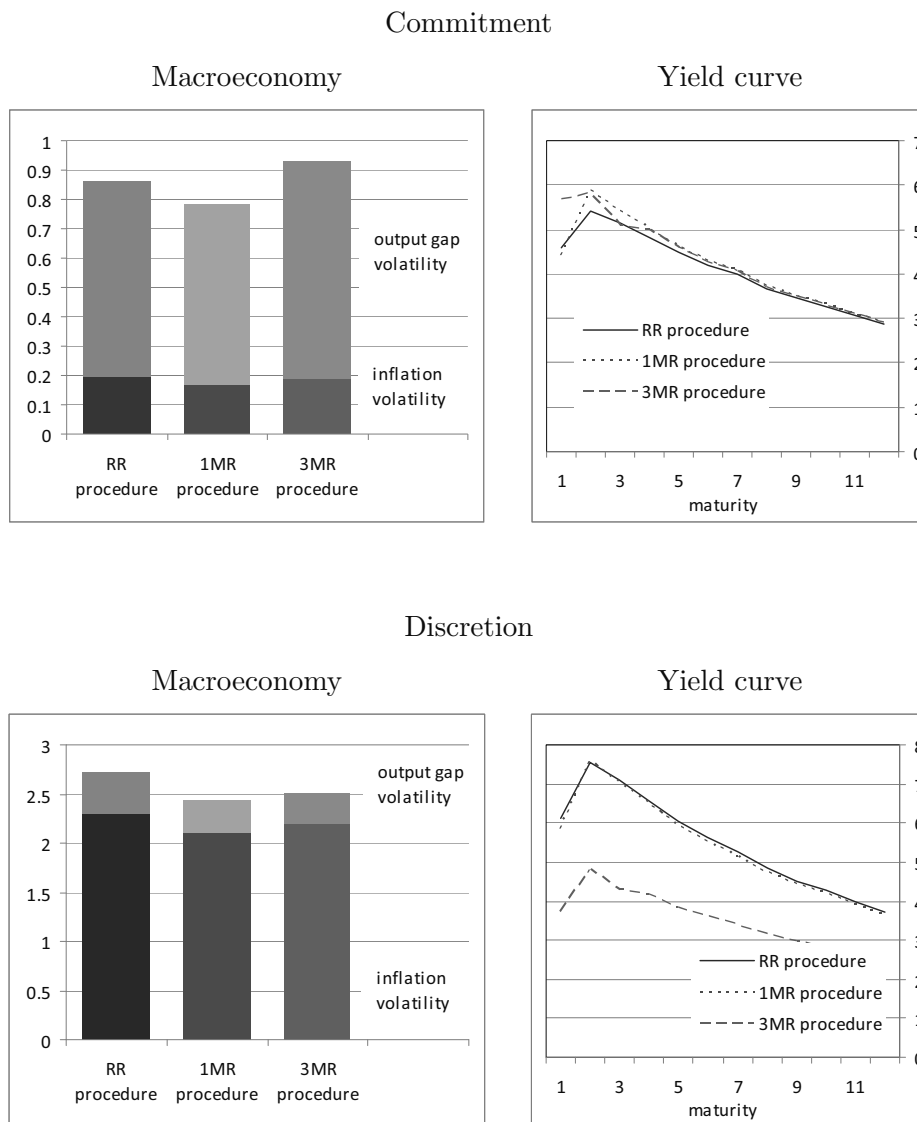
Commitment

Discretion

Note: Simulations with 10,000 draws. RR, 1MR and 3MR stand for
repo rate, one-month and three-month money market rate procedures.

Under discretion, arguably the more realistic assumption for times of financial turmoil, we find in the lower part of Figure 5 a clear increase in macroeconomic volatility under the RR

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procedure and a smaller rise for the 3MR approach. The yield curve volatility is larger, with the smallest increase under the 3MR procedure. Smoothing a longer-term rate seems to dampen movements along the yield curve when there are large shocks affecting money market rates.

The larger volatility generated under 3MR procedure reflects the fact that the one-month market rate enters the IS curve and biases the results in favour of the 1MR procedure. This assumption is based on the standard linearised New Keynesian model of aggregate output underlying this analysis. Central bankers however often argue that economic activity depends on longer term interest rates and thus that special factors affecting the spread between short term and long term rates will also impact on output. Therefore, we repeat the simulations performed so far under the assumption that a basket of interest rates with maturities lasting from one to twelve months are relevant in the IS curve.

4.3   Average interest rate in the IS curve

To examine how the results depend on the choice of the interest rate in the IS curve, we replace the original IS curve (3) in the system with the alternative equation (5) and thus let an unweighted average of the market interest rates with horizons from one to twelve months determine economic activity. Appendix B spells out in detail the adjustments in the model setup.

Optimal repo rate reaction functions: Since now market rates at all horizons impact on the output gap, central banks adjust their repo rate to shocks at any of these horizons. Table 3 shows the optimal reaction functions for the three procedures. The coefficients are broadly similar to the baseline case, though one striking difference is that the degree of interest rate smoothing increases considerably both under commitment and under discretion. The reason for this is that repo rate changes achieve a higher impact on longer-term rates, which depend on the future expected path of i_t, if they are expected to be followed by similar adjustments in the future. This matches the discussions in Goodhart [21] and Woodford [41], who argue that interest rate smoothing might be a result of policymakers' attempt to impact on long-term rates.

Macroeconomic and interest rate volatility: Figure 6 shows the simulated volatilities of inflation, the output gap and the yield curve in times of financial calm and turmoil. In quiet times the volatilities are very similar no matter which interest rate enters the IS curve.

During the financial turmoil, the different procedures now perform equally well under commitment in terms of macroeconomic volatility. This contrasts with the baseline results. In

Table 3: Optimal reaction functions with the average interest rate in the IS curve

Commitment											
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{2,t}$	$\varepsilon_{3,t}$	$\varepsilon_{4,t}$
RR	-0.005	0.060	0.654	0	0	0.003	0.731	-0.024	-0.024	-0.024	-0.024
1MR	-0.015	0.053	0	0.636	0	-0.176	0.778	-0.904	-0.025	-0.025	-0.025
3MR	-0.010	0.036	0	0	0.979	-0.338	0.073	-0.004	-0.004	-1.686	-0.004

	$\varepsilon_{5,t}$	$\varepsilon_{6,t}$	$\varepsilon_{7,t}$	$\varepsilon_{8,t}$	$\varepsilon_{9,t}$	$\varepsilon_{10,t}$	$\varepsilon_{11,t}$	$\varepsilon_{12,t}$	ξ^{PC}_{t-1}	ξ^{IS}_{t-1}
RR	-0.024	-0.024	-0.024	-0.024	-0.024	-0.024	-0.024	-0.024	-0.007	0.095
1MR	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.025	-0.020	0.069
3MR	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.011	0.097

Discretion											
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{2,t}$	$\varepsilon_{3,t}$	$\varepsilon_{4,t}$
RR	0.062	0.115	0.449	0	0	2.809	1.238	-0.048	-0.048	-0.048	-0.048
1MR	0.061	0.118	0	0.436	0	2.555	1.290	-0.808	-0.050	-0.050	-0.050
3MR	0.122	0.284	0	0	0.858	2.791	1.133	-0.069	-0.069	-1.602	-0.069

|	$\varepsilon_{5,t}$	$\varepsilon_{6,t}$	$\varepsilon_{7,t}$	$\varepsilon_{8,t}$	$\varepsilon_{9,t}$	$\varepsilon_{10,t}$	$\varepsilon_{11,t}$	$\varepsilon_{12,t}$
RR	-0.048	-0.048	-0.048	-0.048	-0.048	-0.048	-0.048	-0.048
1MR	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050	-0.050
3MR	-0.069	-0.069	-0.069	-0.069	-0.069	-0.069	-0.069	-0.069

Note: Repo rate reaction function coefficients for different operating procedures, assuming the IS curve is given by equation (5) rather than equation (3). RR, 1MR and 3MR stand for repo rate, one-month and three-month money market rate procedures.

Figure 6: Volatilities with the average interest rate in the IS curve
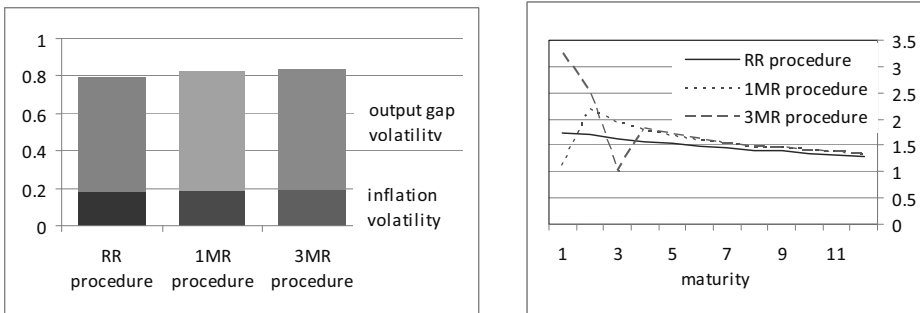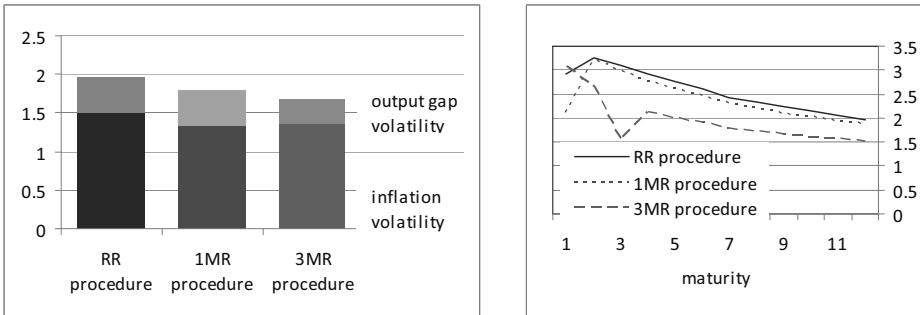
Financial calm: Commitment
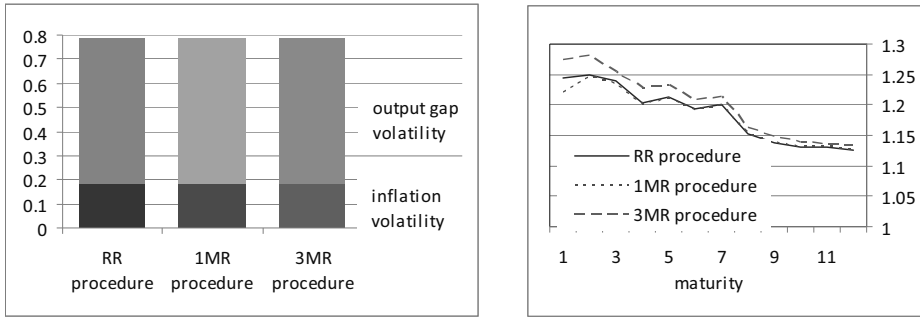
Financial calm: Discretion
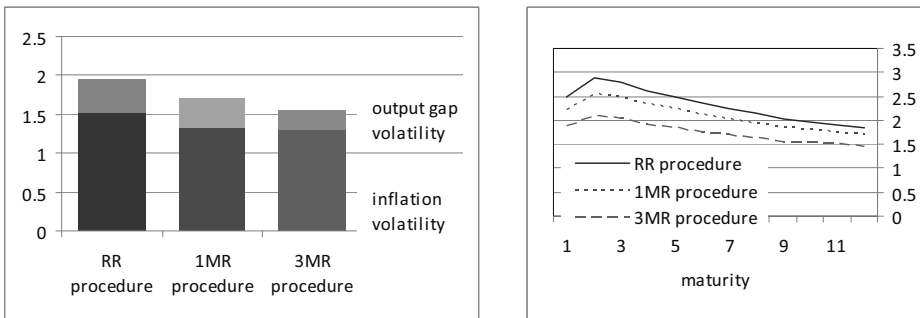
Financial turmoil: Commitment

Financial turmoil: Discretion

Note: Simulations with 10,000 draws. RR, 1MR and 3MR stand for repo rate, one-month and three-month money market rate procedures.

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Figure 5, where we assumed that only the one-month repo rate matters in the IS curve, the 1MR procedure, which smooths that rate and thus responds most aggressively to shocks at that horizon, outperformed the RR and the 3MR approach. Now, smoothing one particular interest rate does not constitute a clear advantage for any of the models, since no single interest rate matters for economic activity. That said, the smoothing objectives show up in the volatility of the yield curve, with little variability in the one-month (three-month) market rates for the 1MR (3MR) procedure. If there is financial turmoil and if monetary policy is set in a discretionary way, the 3MR procedure yields the lowest macroeconomic volatility in Figure 6. This finding again differs from the baseline results, where the 1MR procedure yielded the most stable inflation and output gap. The reason for this change is that the 3MR procedure attaches more weight to the future and therefore implicitly also to the average interest rate that enters the IS curve. At the same time, this approach yields comparatively little yield curve volatility.

In sum, deciding which interest rate matters in the IS curve has important implications for the choice of monetary operating procedure. If one assumes that economic activity is driven by the one-month market rate, using the 1MR procedure seems attractive. If longer-term rates matter for the output gap as well, the 3MR procedure appears most robust in minimising macroeconomic volatility.

4.4 Targeting the yield curve

Having considered how the model changes if we let longer-term interest rates have an effect on the output gap, we now return to the baseline model with the one-month market rate in the IS curve but ask what happens if the central bank dislikes movements of the yield curve as a whole. This seems an interesting exercise in the aftermath of the financial crisis since there have been calls for monetary policy to control financial market prices and asset prices in particular. While our model neither allows us to examine asset prices in detail nor models boom-bust cycles, a drop in the level of the yield curve can be thought of as boosting asset prices. A policy trying to avoid asset price booms should thus prevent shifts in the level of the yield curve.

To analyse this kind of policy, we expand the set of goal variables to include the level of the yield curve. More precisely, policymakers are assumed to minimise the variability of the yield curve around its mean, which is given for the different maturities j by $\theta_j + \tau_j$. We denote the weight attached to the goal of stabilising the yield curve by λ_{yc}. Modelling interest rates up to

twelve months out, $j = 1, 2, ..., 12$, the expanded vector of goal variables is therefore

$$Y_t = \begin{bmatrix} \pi_t & y_t & \Delta i_t & \Delta i_{1,t} & \Delta i_{3,t} & i_{1,t} & ... & i_{12,t} \end{bmatrix}',$$

where the weight attached to each money market rate is $\lambda_{yc}/12$.

Table 4: Optimal reaction functions under yield curve targeting

	Commitment									
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{2,t}$	$\varepsilon_{3,t}$	$\varepsilon_{4,t}$
RR	-0.005	0.068	0.409	0	0	0.234	0.733	-0.340	-0.035	-0.023	-0.017
1MR	-0.013	0.060	0	0.401	0	0.141	0.783	-0.936	-0.034	-0.022	-0.016
3MR	-0.004	0.113	0	0	0.431	0.062	0.543	-0.439	-0.075	-0.764	-0.037

	$\varepsilon_{5,t}$	$\varepsilon_{6,t}$	$\varepsilon_{7,t}$	$\varepsilon_{8,t}$	$\varepsilon_{9,t}$	$\varepsilon_{10,t}$	$\varepsilon_{11,t}$	$\varepsilon_{12,t}$	ξ_{t-1}^{PC}	ξ_{t-1}^{IS}
RR	-0.014	-0.011	-0.010	-0.009	-0.008	-0.007	-0.006	-0.006	-0.015	0.122
1MR	-0.013	-0.011	-0.009	-0.008	-0.007	-0.006	-0.006	-0.005	-0.024	0.092
3MR	-0.030	-0.025	-0.021	-0.019	-0.017	-0.015	-0.014	-0.012	-0.015	0.253

	Discretion									
Procedure	π_{t-1}	y_{t-1}	i_{t-1}	$i_{1,t-1}$	$i_{3,t-1}$	$u_{\pi,t}$	$u_{y,t}$	$\varepsilon_{1,t}$	$\varepsilon_{2,t}$	$\varepsilon_{3,t}$	$\varepsilon_{4,t}$
RR	0.058	0.123	0.242	0	0	5.545	1.855	-0.545	-0.007	-0.005	-0.003
1MR	0.057	0123	0	0.236	0	5.217	1.902	-0.989	-0.006	-0.004	-0.003
3MR	0.090	0.220	0	0	0.244	5.961	1.864	-0.736	-0.019	-0.464	-0.009

	$\varepsilon_{5,t}$	$\varepsilon_{6,t}$	$\varepsilon_{7,t}$	$\varepsilon_{8,t}$	$\varepsilon_{9,t}$	$\varepsilon_{10,t}$	$\varepsilon_{11,t}$	$\varepsilon_{12,t}$
RR	-0.003	-0.002	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001
1MR	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001	-0.001	-0.001
3MR	-0.007	-0.006	-0.005	-0.005	-0.004	-0.004	-0.003	-0.003

Note: Repo rate reaction function coefficients for different operating procedures, $\lambda_{yc} = 1$. RR, 1MR and 3MR stand for repo rate, one-month and three-month money market rate procedures.

Optimal repo rate reaction functions: Table 4 presents the optimal repo rate rules for the three operating procedures assuming $\lambda_{yc} = 1$. Comparing the coefficients with those in Table 1, we see that the expansion of the set of goal variables has little effect on the optimal reaction functions for the RR and 1MR procedures both under commitment and discretion. The main

change is that the goal of stabilising the yield curve increases the number of shocks monetary policy responds to. As a consequence, there now is a reaction not only to the one-month risk premium shock but also to the two-month to twelve-month risk premium shocks. Quantitatively, the response to risk shocks beyond the one-month time horizon is essentially zero.

The repo rate reaction function changes more for the 3MR procedure, both under commitment and discretion. In particular, there is less smoothing of the three-month rate but a more aggressive response to one-month risk shocks. The reason for this is that under yield curve targeting also this procedure attaches some weight to the one-month market rate.

Macroeconomic and interest rate volatility: Figure 7 shows the simulated volatilities of inflation, the output gap and the term structure. To examine the impact of yield curve targeting, we let λ_{yc} increase in a stepwise manner from 0.1 to 1. It can be seen that the variability of the economy rises slightly under commitment and strongly under discretion. While it is perhaps not surprising that the variances of inflation and the output gap become larger when policymakers attempt to achieve an additional goal, it is noteworthy that the variance of the yield curve also increases. This is due to the fact that a more variable output gap raises the volatility of default risk and hence of the yield curve. The effect is stronger under discretion since policymakers take expectations, which drive the yield curve, as given.

5   Conclusions

In this paper, we examine how the choice of monetary operating procedure influences the volatility of inflation, the output gap and the yield curve. Although highly stylised, the three procedures considered are designed to capture key differences between operating frameworks adopted by the Bank of England, the Federal Reserve and the Swiss National Bank. We use a simple New Keynesian model where the implementation tool for monetary policy is the short-term repo rate in all three cases, but the procedures differ in terms of the interest rate targeted by the central bank.

The results suggest that in normal times volatilities resulting from the use of the three procedures are similar under commitment. Under discretion, the macroeconomic volatility is comparatively high if the central bank uses the repo rate as official policy target rate. The procedures based on market rates provide better results in this case. In times of financial distress, with large and highly correlated risk premium shocks affecting market rates, the repo rate

Figure 7: Volatilities under yield curve targeting
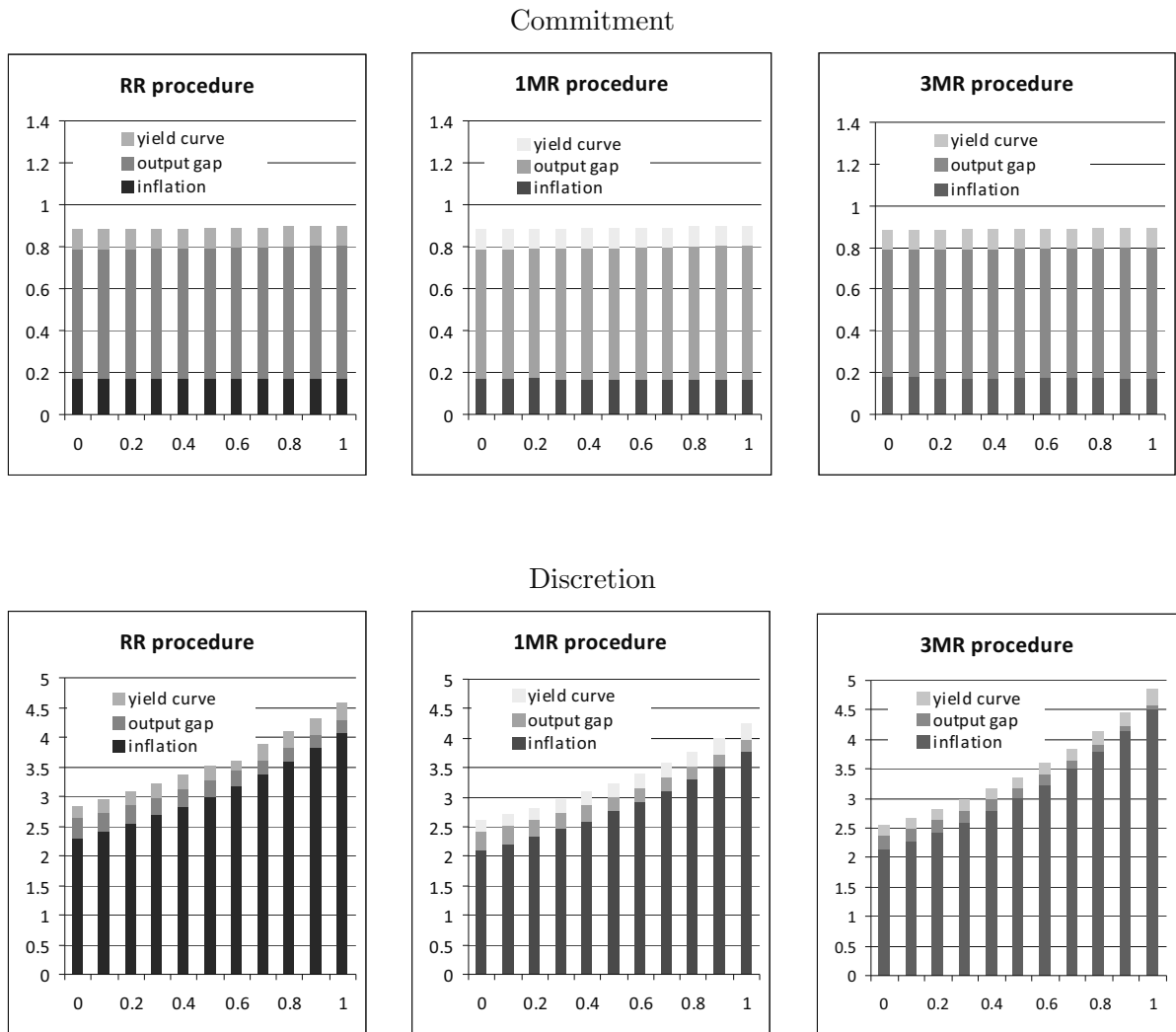
Commitment

Discretion

Note: Simulations with 10,000 draws. RR, 1MR and 3MR stand for
repo rate, one-month and three-month money market rate procedures.
Weight on yield curve variance λ_{yc} increasing from 0 to 1.

procedure again yields high macroeconomic volatility, both under discretion and commitment. Under commitment and large financial shocks, the longer-term money market rate procedure performs poorly in the baseline model. However, this is due to the modelling assumption that the short-term money market rate enters the IS curve. If we instead let the average market rate over horizons of one to twelve months impact on the output gap, the longer-term market rate procedure with its focus on the future becomes attractive. Under discretion, finally, the market-rate based procedures again yield lower macroeconomic volatility than the repo-rate based procedure.

To sum up, none of the three operating procedures studied in this paper is superior in all circumstances. Arguably, this explains why there has not been a convergence to a single operating approach in practice. That being said, it appears that a procedure in which policy targets a money market rate performs best in periods of large shocks to the risk premium such as in the recent past.

A Optimisation with the one-month market rate in the IS curve

A.1 Commitment

The model can be rewritten in state space form,

$$X_{t+1} = A_{10} + A_{11}X_t + A_{12}x_t + B_1 i_t + Ce_{t+1},$$

$$E_t H x_{t+1} = A_{21}X_t + A_{22}x_t + B_2 i_t.$$

where X_t is a vector with $n_X = 31$ predetermined variables

$$\underset{31\times 1}{X_t} = \begin{bmatrix} \pi_{t-1} & y_{t-1} & i_{t-1} & u_{\pi,t} & u_{y,t} & \varepsilon_{1,t} & ... & \varepsilon_{12,t} & \varepsilon_{1,t-1} & ... & \varepsilon_{12,t-1} & i_{1,t-1} & i_{3,t-1} \end{bmatrix}',$$

x_t is a 2×1-vector of forward-looking variables, $x_t = \begin{bmatrix} \pi_t & y_t \end{bmatrix}'$ in period t, the vector i_t is a scalar containing the monetary policy implementation rate, i_t, and e_t is an n_X-vector of white noise innovations to the AR(1) error processes of inflation, the output gap and the risk premia of market interest rates,

$$\underset{31\times 1}{e_t} = \begin{bmatrix} 0 & 0 & 0 & e_{\pi,t+1} & e_{y,t+1} & e_{1,t+1} & ... & e_{12,t} & \mathbf{0}_{1x14} \end{bmatrix}'.$$

Next, we expand the vector of predetermined variables

$$\widetilde{X}_t = \begin{bmatrix} X_t & \Xi_{t-1} \end{bmatrix}',$$

where $\Xi_t = \begin{bmatrix} \Xi_t^{PC} & \Xi_t^{IS} \end{bmatrix}'$ contains the Lagrange multipliers for the Phillips and the IS curve and rewrite the state space system as

$$\widetilde{X}_{t+1} = \widetilde{A}_{10} + \widetilde{A}_{11}\widetilde{X}_t + \widetilde{A}_{12}x_t + \widetilde{B}_1 i_t + \widetilde{C}e_{t+1} \tag{A1}$$

and

$$E_t H x_{t+1} = A_{21}\widetilde{X}_t + A_{22}x_t + B_2 i_t.$$

The matrices $\widetilde{A}_{10}$, $\widetilde{A}_{11}$, $\widetilde{A}_{12}$, $\widetilde{B}_1$, $\widetilde{C}\widetilde{C}'$, H, A_{21}, A_{22} and B_2 are given by

$$\underset{33\times 1}{\widetilde{A}_{10}} = \begin{bmatrix} \mathbf{0}_{1\times 29} & \theta_1 + \tau_1 & \theta_3 + \tau_3 & 0 & 0 \end{bmatrix}',$$

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$$
\widetilde{A}_{11}_{33\times33} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_\pi & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_y & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_\varepsilon & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_\varepsilon & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\varepsilon & \ldots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & \rho_\varepsilon & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{0}_{29\times33} \\
-cf_y M \\
\frac{1}{3}\sum_{k=1}^{2} f_i M^k - \frac{c}{3}\sum_{k=1}^{3} f_y M^k \\
0 \\
0
\end{bmatrix}
$$

(with the elements in the last matrix explained below),

$$
\widetilde{A}_{12}_{33\times2} =
\begin{bmatrix}
1 & 0 & \mathbf{0}_{1\times31} \\
0 & 1 & \mathbf{0}_{1\times31}
\end{bmatrix}',
$$

$$
\widetilde{B}_{1}_{33\times1} =
\begin{bmatrix}
0 & 0 & 1 & \mathbf{0}_{1\times26} & 1 & 1/3 & 0 & 0
\end{bmatrix}'
$$

$$\widetilde{C}\widetilde{C}'_{33\times33} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\pi^2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_y^2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon,1}^2 & \sigma_{\varepsilon,1,2} & \cdots & \sigma_{\varepsilon,1,11} & \sigma_{\varepsilon,1,12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon,1,2} & \sigma_{\varepsilon,2}^2 & \cdots & \sigma_{\varepsilon,2,11} & \sigma_{\varepsilon,2,12} & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon,1,11} & \sigma_{\varepsilon,2,11} & \cdots & \sigma_{\varepsilon,11}^2 & \sigma_{\varepsilon,11,12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\varepsilon,1,12} & \sigma_{\varepsilon,2,12} & \cdots & \sigma_{\varepsilon,11,12} & \sigma_{\varepsilon,12}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} a_\pi & 0 \\ b_r & b_y + b_r c \end{bmatrix},$$

$$\widetilde{A}_{21} = \begin{bmatrix} -(1-a_\pi) & 0 & 0 & -1 & 0 & 0 & \mathbf{0}_{1\times27} \\ 0 & -(1-b_y) & 0 & 0 & -1 & b_r & \mathbf{0}_{1\times27} \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} 1 & -a_y \\ 0 & 1 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 0 \\ b_r \end{bmatrix}.$$

Under operating procedures that steer money market rates, the target rate $i_{1,t}$ or $i_{3,t}$, respectively, contains a risk premium that depends on the expected future path of the output gap. Since y_t is a state variable, these expectations are driven by optimal policy, which thus should be used as an input to the optimisation problem but at the same time is its solution. We solve this problem by assuming starting values for optimal policy and then iterating until convergence. In particular, we define the optimal output gap as

$$y_t = f_y \widetilde{X}_t \tag{A2}$$

and the optimal repo rate as

$$i_t = f_i \widetilde{X}_t. \tag{A3}$$

These definitions are used in setting up the matrix $\widetilde{A}_{11}$ above. There, the risk premium for the one-month rate is given by

$$\theta_{1,t} = \theta_1 - cE_t y_{t+1} + \varepsilon_{1,t} = \theta_1 - cf_y M\widetilde{X}_t + \varepsilon_{1,t},$$

since

$$E_t \widetilde{X}_{t+1} = M\widetilde{X}_t,$$

with M the optimal linear projection matrix defined below. Similarly, the three-month risk premium is

$$\theta_{3,t} = \theta_3 - \frac{c}{j} E_t \sum_{k=1}^{3} y_{t+k} + \varepsilon_{3,t} = \theta_3 - \frac{c}{j} \sum_{k=1}^{3} f_y M^k \widetilde{X}_t + \varepsilon_{j,t},$$

The money market rates in equation (A1) above thus are defined as

$$i_{1,t} = \theta_1 + i_t - cf_y M\widetilde{X}_t + \varepsilon_{1,t}$$

and

$$i_{3,t} = \theta_3 + \frac{1}{3} i_{1,t} + \frac{1}{3} \sum_{k=1}^{2} f_i M^k \widetilde{X}_t - \frac{c}{3} \sum_{k=1}^{3} f_y M^k \widetilde{X}_t + \varepsilon_{3,t},$$

where the first summation sign captures the expectations hypothesis and relies on

$$E_t i_{t+j} = f_i M^j \widetilde{X}_t.$$

To link the goal variables $Y_t = \begin{bmatrix} \pi_t & y_t & \Delta i_t & \Delta i_{1,t} & \Delta i_{3,t} & i_{1,t} & ... & i_{12,t} \end{bmatrix}'$ to the other variables in the model, we define

$$Y_t = D \begin{bmatrix} X_t & \Xi_{t-1} & x_t & \xi_t & i_t \end{bmatrix}',$$

where $n_Y = 17$ and where $\xi_t = \begin{bmatrix} \xi_t^{PC} & \xi_t^{IS} \end{bmatrix}' = \Xi_t$ are Lagrange multipliers that account for the dynamics of the forward-looking variables.

Matrix D is given by

$$
\underset{17\times38}{D} = \left[\begin{array}{ccccccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & \dots & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & \dots & 0 & -1 & 0 & 0 & 0 & 0 & 1/3 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \dots & 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1/3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 & 0 & 0 & 0 & 1/12
\end{array}\right]
$$

$$
+ \left[\begin{array}{c}
0_{3\times38} \\
-cf_yM\mid 0_{1\times5} \\
\frac{1}{3}\sum_{k=1}^{2}f_iM^k - \frac{c}{3}\sum_{k=1}^{3}f_yM^k\mid 0_{1\times5} \\
-cf_yM\mid 0_{1\times5} \\
\frac{1}{2}f_iM - \frac{c}{2}\sum_{k=1}^{2}f_yM^k\mid 0_{1\times5} \\
\vdots \\
\frac{1}{12}\sum_{k=1}^{11}f_iM^k - \frac{c}{12}\sum_{k=1}^{12}f_yM^k\mid 0_{1\times5}
\end{array}\right]
$$

The period loss function in matrix notation is given by

$$
\begin{aligned}
L_t &= \tfrac{1}{2}Y_t'\Lambda Y_t = \tfrac{1}{2}\left[\begin{array}{ccccc} X_t & \Xi_{t-1} & x_t & \xi_t & i_t \end{array}\right]D'\Lambda D\left[\begin{array}{ccccc} X_t & \Xi_{t-1} & x_t & \xi_t & i_t \end{array}\right]' \\
&= \tfrac{1}{2}\left[\begin{array}{ccccc} X_t & \Xi_{t-1} & x_t & \xi_t & i_t \end{array}\right]W\left[\begin{array}{ccccc} X_t & \Xi_{t-1} & x_t & \xi_t & i_t \end{array}\right]'.
\end{aligned}
$$

We solve the model using the dual saddlepoint approach discussed in Marcet and Marimon [28]. We follow Svensson and Williams [38] and define the dual period loss function as

$$
\begin{aligned}
\widetilde{L}_t &= L_t + \Xi_t'(Hx_{t+1} - \widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2i_t) \\
&= L_t + \Xi_t'(-\widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2i_t) + \tfrac{1}{\delta}\Xi_{t-1}'Hx_t \\
&= L_t + \xi_t'(-\widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2i_t) + \tfrac{1}{\delta}\Xi_{t-1}'Hx_t
\end{aligned} \tag{A4}
$$

where the second equality comes from the definition $\Xi_{-1} = 0$. Using equation (13), equation (A4) can be rewritten as

$$
\begin{aligned}
\widetilde{L}_t &= L_t + \xi_t'(-\widetilde{A}_{21}\widetilde{X}_t - A_{22}x_t - B_2 i_t) + \tfrac{1}{\delta}\Xi_{t-1}'Hx_t \\
&= \tfrac{1}{2}\left[\begin{array}{cc} \widetilde{X}_t & \widetilde{i}_t \end{array}\right]\widetilde{W}\left[\begin{array}{cc} \widetilde{X}_t & \widetilde{i}_t \end{array}\right]',
\end{aligned}
\tag{A5}
$$

where

$$
\widetilde{W} = W + \begin{bmatrix}
0 & 0 & 0 & -\widetilde{A}_{21}' & 0 \\
0 & 0 & \tfrac{1}{\delta}H & 0 & 0 \\
0 & \tfrac{1}{\delta}H' & 0 & -A_{22}' & 0 \\
-\widetilde{A}_{21} & 0 & -A_{22} & 0 & -B_2 \\
0 & 0 & 0 & -B_2' & 0
\end{bmatrix}.
$$

Equation (A5) is the quadratic loss function in the optimal regulator problem. The linear transition equation for the predetermined variables is given by

$$
\widetilde{X}_{t+1} = \widetilde{A}_{11}\widetilde{X}_t + \widetilde{B}\widetilde{i}_t + \widetilde{C}e_{t+1},
$$

with

$$
\widetilde{B} = \left(\left[\begin{array}{ccc} \widetilde{A}_{12} & 0_{(n_X+n_x)\times n_x} & \widetilde{B}_1 \end{array}\right] + \begin{bmatrix}
0_{n_X\times n_x} & 0_{n_X\times n_x} & 0_{n_X\times n_i} \\
0_{n_x\times n_x} & I_{n_x\times n_x} & 0_{n_x\times n_i}
\end{bmatrix}\right),
$$

where the identity matrix captures $\Xi_t = \xi_t$. The value function $V(\widetilde{X}_t)$ of the saddlepoint problem is quadratic,

$$
V(\widetilde{X}_t) = [(1-\delta)\widetilde{X}_t'V\widetilde{X}_t + \delta\omega],
$$

where ω is a scalar. The Bellman equation can therefore be written as

$$
(1-\delta)\widetilde{X}_t'V\widetilde{X}_t + \delta\omega = (1-\delta)\max_{\{\xi_t\}_{t\geq 0}} \min_{\{x_t, i_t\}_{t\geq 0}} \left\{\widetilde{L}_t + \delta E_t\left[\widetilde{X}_{t+1}'V\widetilde{X}_{t+1} + \frac{\delta}{1-\delta}\omega\right]\right\}.
$$

Iterating over the resulting Riccati equation yields the optimal solution

$$
\widetilde{i}_t = F\widetilde{X}_t,
\tag{A6}
$$

where

$$
F = -(R + \delta\widetilde{B}'V\widetilde{B})^{-1}(N' + \delta\widetilde{B}'V\widetilde{A}_{11}),
$$

$$
M = \widetilde{A}_{11} + \widetilde{B}F
$$

and

$$
V = Q + \delta\widetilde{A}_{11}'V\widetilde{A}_{11} - (N + \delta\widetilde{B}'V\widetilde{A}_{11})'(R + \delta\widetilde{B}'V\widetilde{B})^{-1}(N' + \delta\widetilde{B}'V\widetilde{A}_{11})
$$

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with

$$\widetilde{W} = \begin{bmatrix} Q & N \\ N' & R \end{bmatrix}$$

partitioned conformably with $\widetilde{X}_t$ and $\widetilde{i}_t$. The optimal rule f_i for the repo rate, equation (A3), is given as the last line in equation (A6) and equation (A2), which captures the dynamics of the output gap, is the second line of equation (A6).

A.2 Discretion

To derive the optimal repo rules under discretion, we define A_{10} as the first n_X elements of $\widetilde{A}_{10}$, A_{11} as the first n_X rows and columns of $\widetilde{A}_{11}$, A_{12} as the first n_X rows of $\widetilde{A}_{12}$, A_{21} as the first n_X columns of $\widetilde{A}_{21}$, B_1 as the first n_X elements of $\widetilde{B}_1$, CC' as the first n_X rows and columns of $\widetilde{C}\widetilde{C}'$ and D as $\widetilde{D}$ without the columns referring to Ξ_{t-1} and ξ_t. We then write the period loss function as

$$L_t = \frac{1}{2} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' W \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} \tag{A7}$$

with

$$W = D'\Lambda D.$$

Under discretion, the repo rate i_t is chosen to minimise equation (A7) subject to

$$\begin{bmatrix} X_{t+1} \\ E_t H x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{10} \\ 0 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} i_t + \begin{bmatrix} C \\ 0 \end{bmatrix} e_{t+1}, \tag{A8}$$

$$i_{t+1} = F_{t+1} X_{t+1} \tag{A9}$$

and

$$x_{t+1} = G_{t+1} X_{t+1}, \tag{A10}$$

where F_{t+1} and G_{t+1} are determined in the optimisation in the next period and are assumed to be known today. Taking expectations, combining equations (A8) to (A10) and solving for x_t yields

$$x_t = \overline{A}_t X_t + \overline{B}_t i_t \tag{A11}$$

with

$$\overline{A}_t = (A_{22} - HG_{t+1}A_{12})^{-1}(HG_{t+1}A_{11} - A_{21})$$

and

$$\overline{B}_t = (A_{22} - HG_{t+1}A_{12})^{-1}(HG_{t+1}B_1 - B_2).$$

From this it follows that

$$X_{t+1} = \widehat{A}_t X_t + \widehat{B}_t i_t + Ce_{t+1}$$

with

$$\widehat{A}_t = A_{11} + A_{12}\overline{A}_t$$

and

$$\widehat{B}_t = B_1 + A_{12}\overline{B}_t.$$

Using equation (A11) in equation (A7) yields

$$L_t = \frac{1}{2} \begin{bmatrix} X_t \\ i_t \end{bmatrix}' \begin{bmatrix} Q_t & N_t \\ N_t' & R_t \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix},$$

where

$$Q_t = W_{XX} + W_{Xx}\overline{A}_t + \overline{A}_t' W_{Xx}' + \overline{A}_t' W_{xx} \overline{A}_t,$$

$$N_t = W_{Xx}\overline{B}_t + \overline{A}_t' W_{xx}\overline{B}_t + W_{Xi} + \overline{A}_t' W_{xi}$$

and

$$R_t = W_{ii} + \overline{B}_t' W_{xx}\overline{B}_t + \overline{B}_t' W_{xi} + W_{xi}' \overline{B}_t.$$

The Bellman equation can be written as

$$\frac{1}{2}[(1-\delta)X_t'V_tX_t + \delta\omega_t] = (1-\delta)\min_{i_t}\left[L_t + \delta E_t \frac{1}{2}\left(X_{t+1}'V_{t+1}X_{t+1} + \frac{\delta}{1-\delta}\omega_t\right)\right].$$

From the first order condition, we obtain

$$F_t = -(R_t + \delta\widehat{B}_t'V_{t+1}\widehat{B}_t)^{-1}(N_t + \delta\widehat{B}_t'V_{t+1}\widehat{A}_t)$$

and

$$G_t = \overline{A}_t + \overline{B}_t F_t,$$

and we denote the corresponding equilibrium functions by F and G. Forecasts of X_t are based on

$$X_{t+1} = MX_t + Ce_{t+1}$$

with

$$M = \widehat{A} + \widehat{B}F,$$

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where $\widehat{A}$ and $\widehat{B}$ are the fixed points of the mapping from $(\widehat{A}_{,t+1},\ \widehat{B}_{t+1})$ to $(\widehat{A}_t,\ \widehat{B}_t)$. The equilibrium function F determines the expected future interest rate that enters A_{11} and D in the iterations that are performed until the model converges. Expected future output gaps are determined by G.

B   Optimisation with the average interest rate in the IS curve

If the IS curve is given by equation (5) rather than (3), we need to adjust matrices H, $\widetilde{A}_{21}$ and B_2 to

$$H = \begin{bmatrix} a_\pi & 0 \\ b_r \frac{1}{12}\sum\limits_{j=1}^{12}\frac{1}{j} & b_y + b_r c \frac{1}{12}\sum\limits_{j=1}^{12}\frac{1}{j} \end{bmatrix},$$

$$\widetilde{A}_{21} = \begin{bmatrix} -(1-a_\pi) & 0 & 0 & -1 & 0 & 0 & 0 & \dots & 0 & \mathbf{0}_{1\times 16} \\ 0 & -(1-b_y) & 0 & 0 & -1 & \frac{1}{12}b_r & \frac{1}{12}b_r & \dots & \frac{1}{12}b_r & \mathbf{0}_{1\times 16} \end{bmatrix}$$
$$+ \begin{bmatrix} 0 \\ \frac{1}{12}\sum\limits_{j=2}^{12}\frac{1}{j}\left[f_i\sum\limits_{k=1}^{j-1}E_t\widetilde{X}_{t+k} - (cf_y + f_\pi)\sum\limits_{k=2}^{j}E_t\widetilde{X}_{t+k}\right] \end{bmatrix}$$

and

$$B_2 = \begin{bmatrix} 0 \\ b_r \frac{1}{12}\sum_{j=1}^{12}\frac{1}{j} \end{bmatrix}.$$

The elements involving $\frac{1}{12}$ derive from the definition of the average real interest rate. The average nominal interest rate is given by

$$\begin{aligned}
i_{av,t} =\ & \tfrac{1}{12}(i_{1,t} + i_{2,t} + \dots + i_{12,t}) \\
=\ & \tfrac{1}{12}(\theta_{1,t} + i_t + \tau_2 + \theta_{2,t} + \tfrac{1}{2}[i_t + E_t i_{t+1}] + \dots + \tau_{12} + \theta_{12,t} + \tfrac{1}{12}[i_t + \sum\limits_{k=1}^{11} E_t i_{t+k}]) \\
=\ & \tfrac{1}{12}(\theta_1 + i_t - cE_t y_{t+1} + \varepsilon_{1,t} \\
& + \tau_2 + \theta_2 + \tfrac{1}{2}[i_t + E_t i_{t+1} - cE_t y_{t+1} - cE_t y_{t+2}] + \varepsilon_{2,t} \\
& + \dots \\
& + \tau_{12} + \theta_{12} + \tfrac{1}{12}\left[i_t + \sum\limits_{k=1}^{11} E_t i_{t+k} - cE_t y_{t+1} - \sum\limits_{k=2}^{12} cE_t y_{t+k}\right] + \varepsilon_{12,t})
\end{aligned}$$

$$= \frac{1}{12}\Big(\theta_1 + i_t - cE_ty_{t+1} + \varepsilon_{1,t}$$
$$+\tau_2 + \theta_2 + \tfrac{1}{2}\Big[i_t - cE_ty_{t+1} + f_i \sum_{k=1}^{1} E_t\widetilde{X}_{t+k} - cf_y \sum_{k=2}^{2} E_t\widetilde{X}_{t+k}\Big] + \varepsilon_{2,t}$$
$$+...$$
$$+\tau_{12} + \theta_{12} + \tfrac{1}{12}\Big[i_t - cE_ty_{t+1} + f_i \sum_{k=1}^{11} E_t\widetilde{X}_{t+k} - cf_y \sum_{k=2}^{12} E_t\widetilde{X}_{t+k}\Big] + \varepsilon_{12,t}\Big)$$
$$= \frac{1}{12}(\theta_1 + \varepsilon_{1,t}) + \frac{1}{12}\sum_{j=1}^{12} \tfrac{1}{j}[i_t - cE_ty_{t+1}]$$
$$+\frac{1}{12}\sum_{j=2}^{12}\left\{\tau_j + \theta_j + \varepsilon_{j,t} + \tfrac{1}{j}\left[f_i \sum_{k=1}^{j-1} M^k\widetilde{X}_t - cf_y \sum_{k=2}^{j} M^k\widetilde{X}_t\right]\right\}$$

For the fourth equality, we note that the expected future values of i_t, y_t and π_t are driven by optimal policy. In particular, we define the inflation rate resulting from the optimisation as

$$\pi_t = f_\pi \widetilde{X}_t.$$

Computing $\pi_{av,t}$ along the same lines as the average nominal interest rate above, we obtain

$$E_t\pi_{av,t+1} = \tfrac{1}{12}E_t(\pi_{1,t+1} + \pi_{2,t+1} + ... + \pi_{12,t+1})$$
$$= \tfrac{1}{12}E_t(\pi_{t+1} + \tfrac{1}{2}[\pi_{t+1} + \pi_{t+2}] + ... + \tfrac{1}{12}\sum_{k=1}^{12}\pi_{t+k})$$
$$= \tfrac{1}{12}f_\pi \sum_{j=1}^{12}\tfrac{1}{j}\left[\sum_{k=1}^{j} M^k\widetilde{X}_t\right]$$

Thus, the average real interest rate is

$$i_{av,t} - E_t\pi_{av,t+1} = \frac{1}{12}(\theta_1 + \varepsilon_{1,t}) + \frac{1}{12}\sum_{j=1}^{12}\frac{1}{j}[i_t - cE_ty_{t+1} - E_t\pi_{t+1}]$$
$$+\frac{1}{12}\sum_{j=2}^{12}\left\{\tau_j + \theta_j + \varepsilon_{j,t} + \frac{1}{j}\left[f_i \sum_{k=1}^{j-1} M^k\widetilde{X}_t - (cf_y + f_\pi)\sum_{k=2}^{j} M^k\widetilde{X}_t\right]\right\}.$$

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