Has the American Output Growth Path Experienced a Permanent Change?

Thomas Lustenberger

SNB Working Papers
14/2018

SCHWEIZERISCHE NATIONALBANK
BANQUE NATIONALE SUISSE
BANCA NAZIONALE SVIZZERA
BANCA NAZIUNALA SVIZRA
SWISS NATIONAL BANK

Legal Issues

DISCLAIMER

The views expressed in this paper are those of the author(s) and do not necessarily represent those of the Swiss National Bank. Working Papers describe research in progress. Their aim is to elicit comments and to further debate.

COPYRIGHT©

The Swiss National Bank (SNB) respects all third-party rights, in particular rights relating to works protected by copyright (information or data, wordings and depictions, to the extent that these are of an individual character).

SNB publications containing a reference to a copyright (© Swiss National Bank/SNB, Zurich/year, or similar) may, under copyright law, only be used (reproduced, used via the internet, etc.) for non-commercial purposes and provided that the source is mentioned. Their use for commercial purposes is only permitted with the prior express consent of the SNB.

General information and data published without reference to a copyright may be used without mentioning the source. To the extent that the information and data clearly derive from outside sources, the users of such information and data are obliged to respect any existing copyrights and to obtain the right of use from the relevant outside source themselves.

LIMITATION OF LIABILITY

The SNB accepts no responsibility for any information it provides. Under no circumstances will it accept any liability for losses or damage which may result from the use of such information. This limitation of liability applies, in particular, to the topicality, accuracy, validity and availability of the information.

ISSN 1660-7716 (printed version)
ISSN 1660-7724 (online version)

© 2018 by Swiss National Bank, Börsenstrasse 15,
P.O. Box, CH-8022 Zurich

Has the American Output Growth Path Experienced a Permanent Change?*

Thomas Lustenberger[a]
Swiss National Bank and University of Basel

November 1, 2018

Abstract

In this paper, I derive and apply three univariate methods and one bivariate method to estimate permanent and transitory components of the American output growth path during the 1790 to 2017 period. The results show that statistical tests give little support to the hypothesis of significant permanent growth rate changes (univariate methods). The "special century" (1870-1970, as defined by Gordon (2016)) exhibited more volatile permanent shifts in the output level compared to recent decades (bivariate method).

JEL Classification: E32, E47
Keywords: Output growth, business cycle, permanent and transitory components

1 Introduction

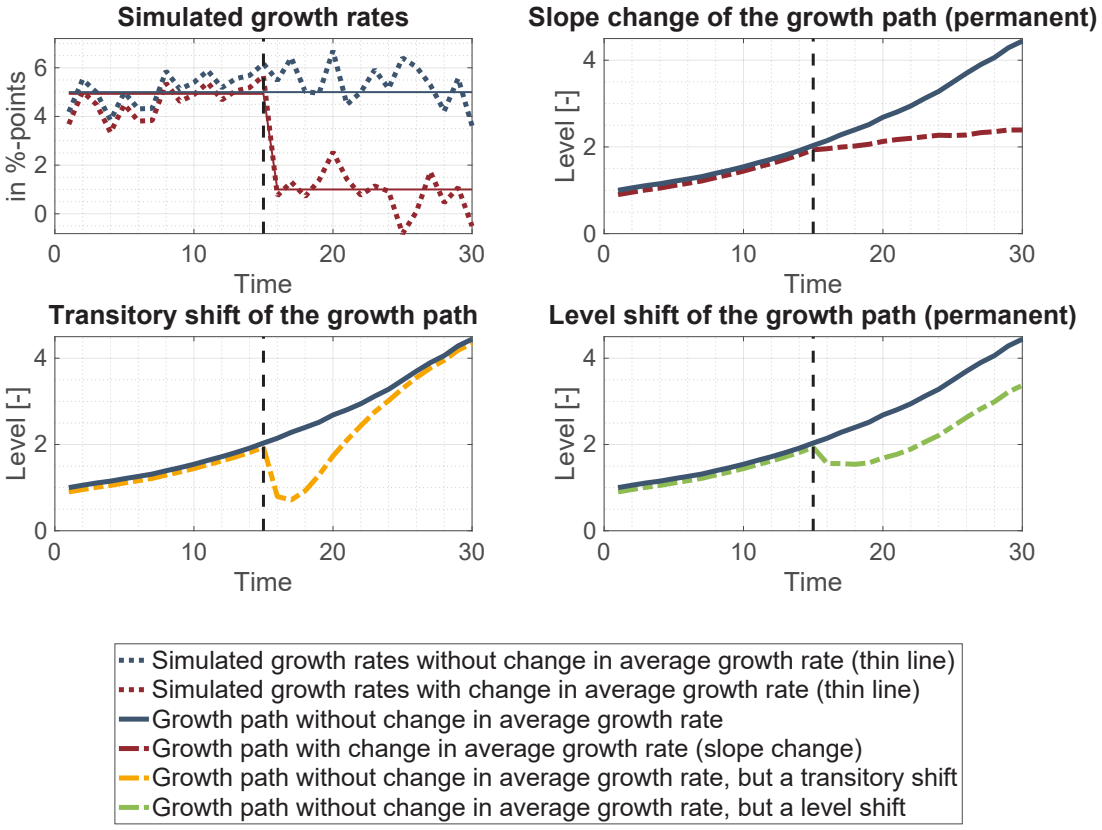
The output growth path determines welfare in an economy. Welfare is often interpreted as the standard of living.[1] A growth path is affected by its growth rate and its initial value. Throughout this paper, I refer to the average growth rate as the growth path slope, while the initial value (or current value) is referred to as the growth path level. Both the growth path slope and the growth path level can change permanently over time, determining the growth path.

*The author thanks Angela Abbate, Anna Tina Campell, Peter Lustenberger, Cyril Monnet, Enzo Rossi, Marcel Savioz, Heinz Zimmermann and an anonymous referee for their valuable comments and suggestions. In addition, the paper benefited from comments by the participants of the SNB Brown Bag Seminar and the SSES Annual Congress 2018. The views, opinions, findings, and conclusions or recommendations expressed in this paper are strictly those of the author(s). They do not necessarily reflect the views of the Swiss National Bank (SNB). The SNB takes no responsibility for any errors or omissions in, or for the correctness of, the information contained in this paper.

[a]thomas.lustenberger@snb.ch

[1] Standard of living is often approximated by welfare per person. Output measures such as GDP or GNP are proxies for welfare. See, for instance, Gordon (2016).

Figure 1: Stylized transitory and permanent changes in the output growth path



The upper left-hand figure shows simulated growth rates. The dotted blue line is a simulation with a 5% average growth rate over the whole sample. The dotted red line shows simulations with, first, an average growth rate of 5% until period 15 (similar to the blue line) and then with an average growth rate of 1% (the change is indicated by the dashed black line). The upper right-hand figure shows the growth path according to the simulated growth rates from the upper left-hand figure. The blue line shows the growth path without any change in the average growth rate, while the dash-dot red line shows the growth path with a change. This change is a slope change in the growth path (permanent). The lower left-hand figure plots a transitory shift (dash-dot orange line) in period 15, while the lower right-hand figure shows a level shift (dash-dot green line) in period 15, as indicated by the dashed black line. Following Blanchard & Quah (1989), a transitory shift in the growth path is associated with a transitory disturbance, while a level shift in the growth path refers to a permanent disturbance.

A permanent change in the growth path slope can heavily affect future output outcomes. An average growth rate of 5% per year leads to a doubling of the growth path level compared to an average growth rate of 1% per year over a period of 15 years. Obviously, a permanent change in these growth rates, such as a shift in average growth from 5% to 1%, determines the growth path. The upper left-hand and right-hand panels of **Figure 1** illustrate this fact.

A permanent change in the growth path level (level shift) at a certain point in time affects the future output growth path. For example, a negative permanent change shifts the growth path level downwards, and consequently, the economy cannot maintain the growth path level that would have been reached without this shift. The lower right-hand panel of **Figure 1** shows such a shift. Such a permanent change also affects the growth path of an economy. Blanchard & Quah (1989) developed a method to identify such level shifts, which they refer to as permanent disturbances. For completeness, the lower left-hand panel of **Figure 1** plots the effect of a transitory shift on the output growth path. Blanchard & Quah (1989) call these transitory disturbances.

As illustrated, both types of permanent change – a slope change and a level shift (upper and lower right panels in **Figure 1**) – affect the future output growth path. For instance, a negative permanent change means that incomes are lower and that companies invest less and hire fewer people. Knowledge about such permanent changes allows the output growth path – and, consequently, the well-being of society – to be better predicted.

Innovation is the main driver of output growth. Gordon (2016) states that a permanent change occurred in US output growth – a slope change – in the 1970s. He claims that the kind of rapid output growth experienced in the "special century" from 1870 to 1970 was a one-off event because there were simply not enough new innovations that could compete with the innovations made during this "special century". For instance, he remarks that the invention of the light bulb by Thomas Edison is difficult to beat. In addition, Gordon (2016) argues that real GDP per capita understates the improvement in the standard of living during this period. GDP misses many dimensions that are highly valued by people when they measure their quality of life. For example, people may value access to a toilet and piped water more than access to the Internet. Gordon (2016) believes that major innovations to improve the standard of living have taken place. Thus, he predicts future output growth to be +0.2% p.a. on average.[2] Compared to the average growth rate in the last two centuries of about 3.5% p.a.,[3] this figure constitutes a permanent slope change in the output growth path.

[2] See, for instance, `https://www.ted.com/talks/robert_gordon_the_death_of_innovation_the_end_of_growth?`, viewed in August 2017.
[3] See **Table 1**.

Has there been a permanent change in the American output growth path? To answer this question, I perform two exercises – one related to slope changes and the other related to level shifts. In the first exercise, I estimate average growth rates and execute statistical tests to investigate possible permanent changes in the growth path slope over time. As robustness checks, I estimate permanent components of output growth in two additional frameworks. Within these frameworks, a significant change in the permanent component is equal to a slope change in the output growth path. a) I apply the Hodrick-Prescott filter (HP filter) to derive the permanent component. I also calculate confidence bands for those estimates following Giles (2012) suggestion. b) I use survey forecasts to estimate permanent components. In addition, the cross-sectional dispersion of forecasts is used as narrow confidence bands. In the second exercise, I apply the procedure suggested by Blanchard & Quah (1989) to estimate permanent disturbances in a bivariate framework. Within this framework, a permanent change (permanent disturbance) constitutes a level shift in the output growth path.

To this end, I use time series of the US economic output beginning in 1790, although survey forecasts started only in the late 1960s. I also collected American unemployment rates beginning in 1869. The unemployment rates are incorporated in the bivariate framework by Blanchard & Quah (1989).

This paper presents four novelties. First, I perform statistical tests on average growth rates and growth rates per capita from 1790 to 2017 (a very long period) and subsamples relating to Gordon (2015) and Gordon (2016). Second, I use confidence bands when applying the HP filter. Third, I use survey forecasts to estimate average growth rates, including the use of their cross-sectional dispersion as narrow confidence bands. And finally, I apply the procedure by Blanchard & Quah (1989) to data beginning in 1869.

The results indicate that there was no significant change in either yearly or quarterly average growth rates and, thus, no significant slope change in the output growth path. In addition, robustness checks support this conclusion. Permanent disturbances à la Blanchard & Quah (1989) – associated with level shifts in the output growth path – are more volatile prior to the Second World War compared to the post-war period. Thus, more severe level shifts in both directions (positive and negative) seem to be an important feature of Gordon's "special century" from 1870 to 1970.

Is Gordon (2016) correct when he claims that the average output growth (slope) has decreased in recent decades? Overall, the hypothesis of a permanent change in the output growth path finds no statistical support. Although it is possible that a permanent change occurred, it was not large enough to be statistically significant. Furthermore, the level shift (permanent) observed during the Great Recession (2008-2009) was the largest in recent decades. However, compared to the pre-war period, this shift seems rather small. In addition, Gordon's prediction of US growth of approximately +0.2% for the decades to come cannot be validated today. Only time will tell what the future holds. Quoting Paul Krugman seems appropriate: "Is he [Gordon] right? My answer is a definite maybe."[4]

This paper is structured as follows. **Section 2** provides a literature review and **Section 3** describes the data. **Section 4** estimates average growth rates over time (slope changes), while **Section 5** adds to robustness. **Section 6** elaborates on a bivariate time series approach to estimate permanent level shifts. **Section 7** concludes the paper.

2 Literature review

In this section, I provide a short literature review. The literature on output and economic growth is extensive. Therefore, I focus on only two types of studies, which are selected based on their statistical methods. The first type includes univariate time series models, while the second type refers to bivariate time series models. In this paper, I use univariate methods to estimate slope changes, while a bivariate method is applied to investigate level shifts. **Figure 1** presents the distinction of slope changes and level shifts in a stylized manner.

2.1 Literature on univariate time series models

The literature proposes several procedures for the univariate decomposition of transitory and permanent components. Some examples include Beveridge & Nelson (1981), Nelson & Plosser (1982), Campbell & Mankiw (1987a,b, 1989) and Cochrane (1988). These examples apply three methods to decompose permanent from transitory components.

[4] Krugman, P. (2016, January 25), "Paul Krugman Reviews 'The Rise and Fall of American Growth' by Robert J. Gordon", *The New York Times*. Retrieved from http://www.nytimes.com.

The first method makes use of so-called unobserved components models. Beveridge & Nelson (1981) introduce a procedure for decomposing non-stationary time series into permanent and transitory components. Nelson & Plosser (1982) also decompose output into a growth component and a cyclical component using an unobserved components model. They find that shocks to the growth component contribute substantially to the variation in observed output. Thus, they associate shocks to the growth component with real disturbances.

The second method, which estimates autoregressive-moving-average (ARMA) models, helps to determine permanent and transitory components. Campbell & Mankiw (1987a,b, 1989) apply an ARMA approach using the change in log GNP to calculate impulse response functions. They find that a 1% innovation in real GNP increases the univariate forecast of GNP by over 1% over any foreseeable horizon (up to 80 quarters, i.e., 20 years). "If fluctuations in output are dominated by temporary deviations from the natural rate of output, then an unexpected change in output today should not substantially change one's forecast of output in, say, five or ten years."

In the third method, Cochrane (1988) uses a nonparametric approach. His idea is that the behaviour of covariances dies out in stationary processes, while they are persistent in non-stationary processes. The idea of Cochrane (1988) is to estimate k-Variances of log GNP to its lag k and plot them against k. i) If the data follow trend stationarity, the plot declines towards zero; ii) if the data follow a random walk, the plot should stay constant; iii) if the data follow a mixture of permanent and temporary components, the plot goes to the random walk variance, i.e., it decreases but stays positive at the random walk variance. Cochrane (1988) finds little long-term persistence in GNP.

However, the (likely) most often used decomposition is currently the Hodrick-Prescott filter (HP filter, see Hodrick & Prescott (1997)). This filter decomposes univariate time series into a trend component (permanent) and a cyclical component (transitory) using a quadratic form. Giles (2012) shows how to construct confidence bands for the permanent component of the HP filter.

Hamilton (2017) criticizes the HP filter while pointing to three issues: i) Series generated by the HP filter exhibit spurious dynamic relations that are not based on the underlying data-generating process. ii) Filtered values in the middle are

different from those at the sample start and end. iii) A statistical formalization of the problem typically produces values for the smoothing parameter (λ) that differ from common practice. Hamilton (2017) thus proposes an alternative: Regress a quarterly time series at $t + h$ on the four most recent values as of date t. Then, produce forecasts using the estimated model and calculate forecast errors. This approach meets all the objectives that users of the HP filter require, but with none of its drawbacks. Hamilton's method is in the spirit of Campbell & Mankiw (1987a,b, 1989) in the sense that a permanent change should lead to persistent changes in forecasts.

Another method that is similar to Hamilton's method is to use survey forecasts as permanent component estimates. For instance, Coibion et al. (2017) use long-run forecasts from *Consensus Forecasts* to approximate the growth rates of potential output. The difference between actual and potential output indicates where we are on the cycle component. Coibion et al. (2017) report that "estimates of potential output have been systematically revised downward since the Great Recession". However, their general conclusion is that an "[a]bsence of clear ways to precisely estimate potential output in real-time suggests that the practice of relying on 'judgement' by professional economists should not be discontinued anytime soon."

In **Section 4**, I estimate average growth rates. In **Section 5**, I present two robustness checks. First, I calculate the permanent components using the HP filter and add confidence bands to the HP filtered time series. Second, I use survey forecasts to estimate permanent components of output growth and add the cross-sectional dispersion as narrow confidence bands.

2.2 Literature on bivariate time series models

The most famous example of a bivariate model of permanent and transitory components of output is Blanchard & Quah (1989). These authors develop a bivariate time series model called vector autoregression (VAR), using real GNP growth and the unemployment rate. They implement a structure for the VAR as follows: They assume two kinds of disturbances, each uncorrelated with the other, and neither has a long-run effect on unemployment. They also assume, however, that supply disturbances have a long-run effect on output, while demand disturbances do not. Their empirical finding is that output fluctuations are driven mainly by demand disturbances (no permanent effect).

Lippi & Reichlin (1993) criticized Blanchard & Quah (1989) because their VAR estimation is based on an arbitrary assumption about the moving-average representation; namely, the assumption that the determinant of the moving-average matrix polynomial is of a modulus less than one. Blanchard & Quah (1993) agree with Lippi & Reichlin (1993), but they (Blanchard & Quah (1993)) point out that Lippi & Reichlin (1993) derived similar quantitative results when adjusting the model according to their criticism.

Keating & Nye (1998) estimate a model in the spirit of Blanchard & Quah (1989), using data for ten developed countries in the nineteenth and twentieth centuries. Observations for the US begin in 1870. In contrast to Blanchard & Quah (1989), Keating & Nye (1998) use the change in the price level instead of the unemployment rate. They find that the Blanchard & Quah (1989) model is not supported by nineteenth-century data.

A second example of a bivariate model that estimates the permanent component in output growth is Cochrane (1994), who estimates two bivariate VARs – GNP-Consumption and Stock Returns-Dividend. Both include their ratios as the independent variable since they exhibit high predictive power. He subsequently finds that disturbances to GNP are almost entirely transitory.

In **Section 6**, I apply the procedure by Blanchard & Quah (1989) to long time series beginning in 1869. In contrast, Blanchard & Quah (1989) use only post-war data. To the best of my knowledge, such long time series of output and unemployment in the Blanchard & Quah (1989) framework have not been used previously.

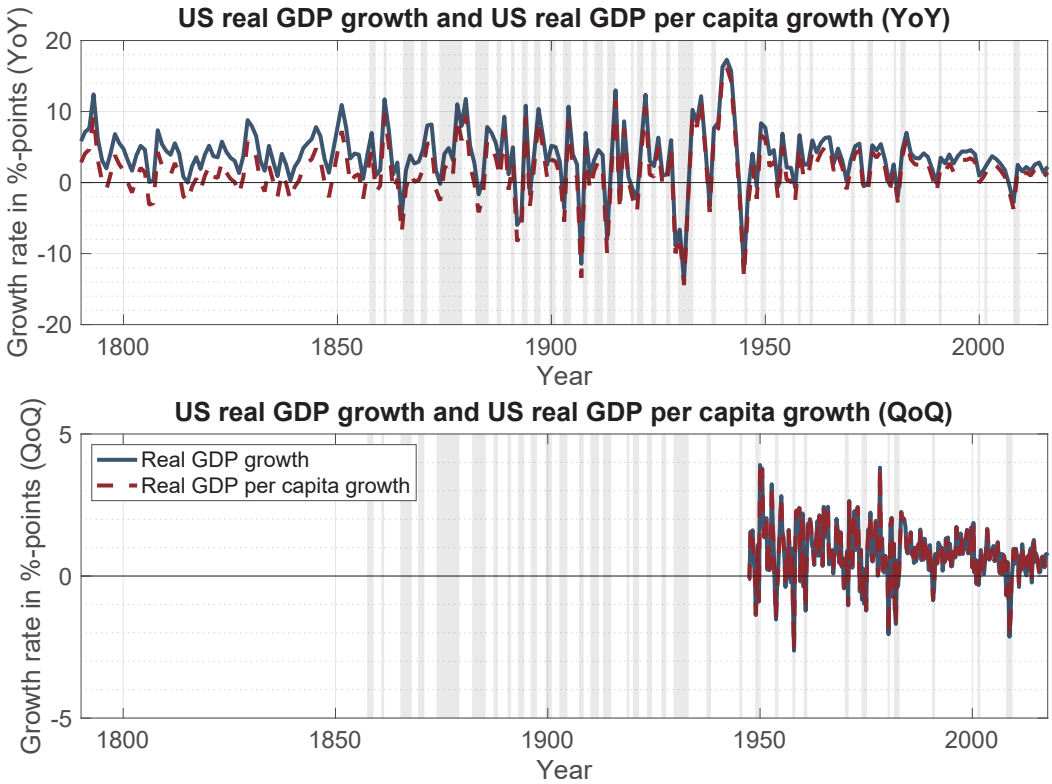
3 Data on output

In this section, I describe the data sources for quarterly and yearly real GDP and real GDP per capita time series.

Yearly US real GDP and US real GDP per capita time series data are from 1790 to 2017. Data are available in millions of 2009 dollars (i.e., level data).[5] The growth

[5] The series are from `https://www.measuringworth.com/usgdp/`, downloaded in June 2018. The series are called *Real GDP (millions of 2009 dollars)* and *Real GDP per capita (year 2009 dollars)*.

Figure 2: Realized real output growth and real output per capita growth (1790–2017)



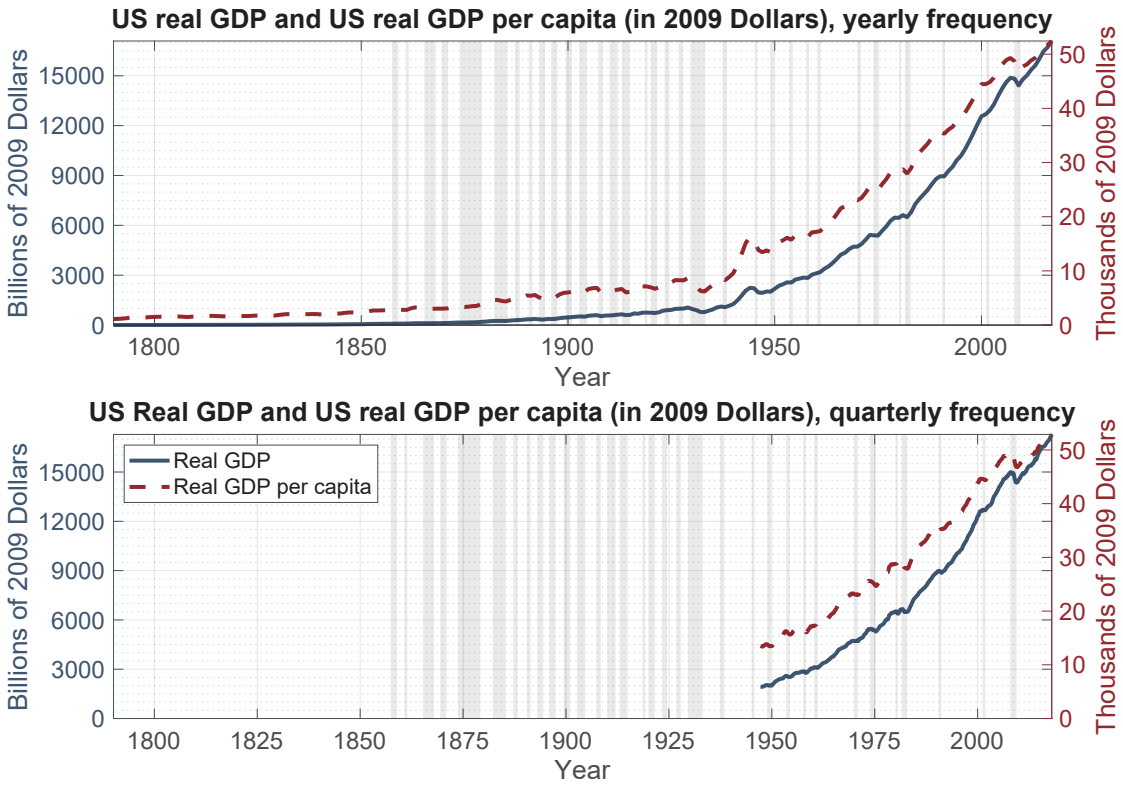
The figure plots realized US real GDP growth and US real GDP per capita growth at a yearly frequency starting in 1790 (upper) and realized US real GDP growth and US real GDP per capita growth at a quarterly frequency beginning in Q1-1947 (lower). Grey shaded areas are the NBER recessions.

rates are calculated in log differences $(\log[y_t] - \log[y_{t-1}]) \cdot 100$ with y_t as real GDP in millions of 2009 dollars in year t or real GDP per capita in 2009 dollars in year t. In addition, I use quarterly US real GDP and US real GDP per capita from Q1-1947 to Q4-2017.[6] I use the quarterly series from Q1-1947 as an explicit series for the post-war period.

Figure 2 plots yearly and quarterly growth series including the National Bureau of Economic Research (NBER) recessions shaded in grey. The NBER classification of recessions begins in 1854. **Figure 2** (upper plot) reveals that yearly growth rates are much less volatile in the post-war period (after 1947). In addition, the frequency of economic downturns decreased over time. Recessions are less fre-

[6] FRED Ticker: GDPC1, called *Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate* and FRED Ticker: A939RX0Q048SBEA, called *Real gross domestic product per capita, Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate*. Both downloaded in June 2018.

Figure 3: Realized real output (1790–2017)



The figure plots realized US real GDP (lhs) and US real GDP per capita (rhs) at a yearly frequency beginning in 1790 (upper) and realized annualized US real GDP (lhs) and US real GDP per capita (rhs) at a quarterly frequency beginning in Q1-1947 (lower). Grey shaded areas are the NBER recessions.

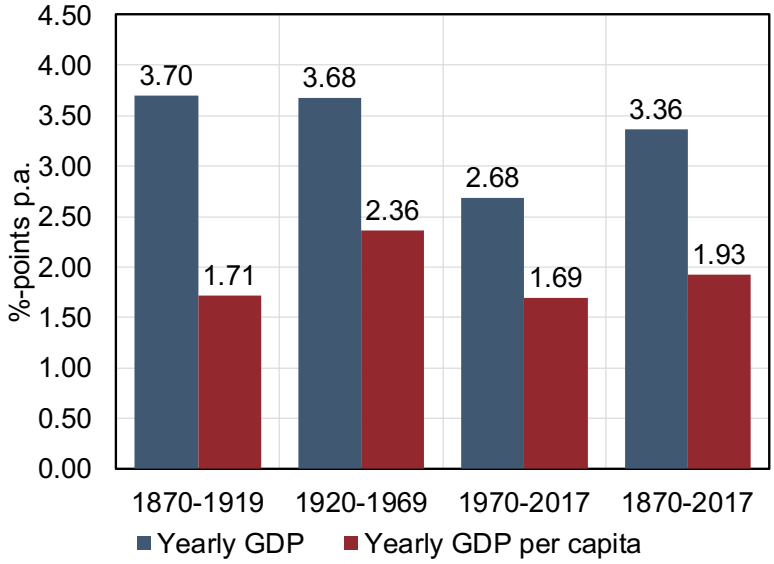
quent in the post-war period than in the pre-war period, which indicates that in recent decades, business cycles tend to be longer than they were prior to 1947.

Figure 3 plots the output level. The grey shaded areas indicate recessions identified by NBER. The output level steadily increases over time, and spikes in the output growth path are rare. The output level per capita behaves very similarly.

4 Estimating slope changes: Average growth rates

In this section, I calculate average growth rates using the time intervals of Gordon (2016), p. 14 and Gordon (2015). Moreover, I test whether a significant change in average growth rates occurred during the sub-periods. A significantly different average growth rate estimate would indicate a slope change of the output growth path.

Figure 4: Average real output growth rates (1870–2017)



The figure shows average real GDP growth rates (blue) and average real GDP per capita growth rates (red) for the US at a yearly frequency. Sub-samples are as in Gordon (2016).

Gordon (2016) distinguishes three sub-periods, from 1870–1919, 1920–1969, and 1970–2014. In contrast, the five sub-periods identified by Gordon (2015) are 1920–1949, 1950–1971, 1972–1995, 1996–2003 and 2004–2016. If there is a permanent change in output growth, the average growth rate should change significantly between the sub-periods. I test whether this was indeed the case by running a regression with time dummies for sub-periods. I refrain from executing structural break tests because doing so would go beyond the scope of this paper.

Figure 4 plots the average growth rates for the sub-periods (in Gordon (2016)) and the overall sample for real GDP growth and real GDP-per-capita growth at a yearly frequency. The average growth rate declined over time from almost +3.7% points p.a. to around +2.7% points p.a. Over the last 150 years, this figure amounts to approximately +3.4% points p.a. In contrast, real GDP-per-capita growth went from +1.7% points p.a. to +2.4% points p.a. before decreasing to +1.7% points p.a. on average.

Regression results are shown in **Table 1**. Panel A shows the sub-periods according to Gordon (2016). For both yearly and quarterly frequency data, the dummy for the sub-period from 1970–2017 is not significant. Therefore, average growth did not change significantly over time. The same is the case for the regressions

using real GDP per capita growth. These results do not support the hypothesis of Gordon (2016) that there has been a slowdown in output growth in recent decades (a decrease in the slope of the output growth path).

Table 1: Average growth rates of output and output per capita (1870–2017)

Panel A: Sample periods as in Gordon (2016)

	Yearly real GDP growth					Yearly real GDP per capita growth				
	Estimate	**t-Stat**	**95% CI**		**% p.a.**	**Estimate**	**t-Stat**	**95% CI**		**% p.a.**
1870–1919	0.0370*	5.15	0.0229	0.0511	3.70	0.0171*	2.36	0.0029	0.0313	1.71
1920–1969	-0.0003	-0.02	-0.0227	0.0222	3.68	0.0065	0.56	-0.0160	0.0290	2.36
1970–2017	-0.0102	-1.32	-0.0253	0.0050	2.68	-0.0002	-0.02	-0.0155	0.0151	1.69
1870–2017 average in % p.a.					**3.36**					**1.93**

	Quarterly real GDP growth					Quarterly real GDP per capita growth				
	Estimate	**t-Stat**	**95% CI**		**% p.a.**	**Estimate**	**t-Stat**	**95% CI**		**% p.a.**
1947–1969	0.0099*	6.36	0.0069	0.0130	4.04	0.0061*	3.79	0.0029	0.0092	2.45
1970–2017	-0.0032	-1.86	-0.0067	0.0002	2.71	-0.0018	-1.03	-0.0053	0.0017	1.71
1947–2017 average in % p.a.					**3.13**					**1.94**

Panel B: Sample periods as in Gordon (2015)

	Yearly real GDP growth					Yearly real GDP per capita growth				
	Estimate	**t-Stat**	**95% CI**		**% p.a.**	**Estimate**	**t-Stat**	**95% CI**		**% p.a.**
1920–1949	0.0328*	2.26	0.0043	0.0613	3.28	0.0212	1.45	-0.0074	0.0497	2.12
1950–1971	0.0075	0.48	-0.0230	0.0380	4.03	0.0041	0.26	-0.0264	0.0347	2.53
1972–1995	-0.0022	-0.14	-0.0321	0.0277	3.06	-0.0009	-0.06	-0.0309	0.0291	2.02
1996–2003	0.0004	0.02	-0.0295	0.0302	3.32	0.0013	0.08	-0.0285	0.0310	2.24
2004–2017	-0.0148	-0.97	-0.0445	0.0150	1.81	-0.0113	-0.74	-0.0411	0.0185	0.99
1920-2017 average in % p.a.					**3.19**					**2.03**

	Quarterly real GDP growth					Quarterly real GDP per capita growth				
	Estimate	**t-Stat**	**95% CI**		**% p.a.**	**Estimate**	**t-Stat**	**95% CI**		**% p.a.**
1947–1949	0.0045	1.34	-0.0021	0.0110	1.80	0.0001	0.03	-0.0064	0.0066	0.04
1950–1971	0.0056	1.53	-0.0016	0.0127	4.08	0.0062	1.70	-0.0010	0.0133	2.55
1972–1995	0.0032	0.89	-0.0038	0.0102	3.09	0.0049	1.38	-0.0021	0.0119	2.03
1996–2003	0.0040	1.11	-0.0030	0.0110	3.42	0.0056	1.59	-0.0013	0.0126	2.32
2004–2017	0.0000	0.01	-0.0069	0.0069	1.81	0.0024	0.67	-0.0045	0.0092	0.99
1947–2017 average in % p.a.					**3.13**					**1.94**

Panel A shows the regression $(\log[y_t] - \log[y_{t-1}]) = b + b_{1920-1969} + b_{1970-2017} + e_t$ where the b's with subscripts stand for dummies of the corresponding time period. Thus, b is the average growth rate from 1870 to 1919 (with similar quarterly data, but beginning in Q1-1947). The average growth rate between 1920 and 1969 is $b + b_{1920-1969}$, whereas the average growth rate between 1970 and 2017 is equal to $b + b_{1970-2017}$. Similarly, Panel B shows estimates for sub-periods as in Gordon (2015). The derivation of average growth rates is the same as in Panel A. The average in % p.a. is the average growth rate p.a. of the observed data. I use Newey-West standard errors with bandwidth 1 for yearly frequency data and 4 for quarterly frequency data. 95% CI shows the confidence intervals. * $p < 0.05$.

Table 1, Panel B shows estimates for sub-periods as in Gordon (2015). Using these sub-periods, the estimated average growth rates are very similar to those of Gordon (2015). However, only the intercept for the yearly frequency is significant at the 5% level, as all the other estimates are not significantly different from zero.

To find a significant decrease in mean growth rates using yearly data, the slope shift needs to be larger than approximately −2.53% points p.a. For quarterly data, the figure is roughly the same (−2.65% points p.a.).[7] Therefore, as long as growth rates do not fall from +3.5% points p.a. to below +1.0% points p.a. on average, it is impossible to detect a significant change, given that the standard deviation remains unchanged. Put differently, as long as American growth rates do not rise to Indian or Chinese levels,[8] the applied statistical tests cannot detect slope shifts.

To sum up, empirical results do not significantly support the hypothesis of Gordon (2016) that there has been a slowdown in output growth. There is no statistically significant permanent change in average output growth rates.

5 Estimating slope changes: Robustness using two additional methods

Given that in the previous section, I found no significant change in the slope, I apply two additional methods to support this result. First, I apply the two-sided HP filter and calculate confidence bands for permanent components estimates (slope) by applying a method suggested by Giles (2012). Second, I use survey forecasts to estimate permanent components. This method is similar to Coibion et al. (2017). Survey forecasts have the advantage that the information set incorporated in forecasts is much larger than that, for example, in the method suggested by Hamilton (2017), and there is no need to account for structural breaks – this is performed by professional forecasters. In addition to Coibion et al. (2017), I approximate confidence bands of the mean forecasts using the cross-sectional standard deviation of reported forecasts. In both the HP filter

[7] For quarterly data (Panel A, left-hand side), this is approximately $[(1-0.0067)^4-1] = -0.0265$, i.e., −2.65% points p.a.

[8] See, for example, the World Economic Outlook April 2018 published by the IMF. The U.S. had a growth rate of +2.5%, while India had +6.7% and China +6.9%

and survey forecasts, a significantly different permanent component estimate (or forecast) is associated with a slope change in the output growth path.

5.1 HP filter with a confidence band

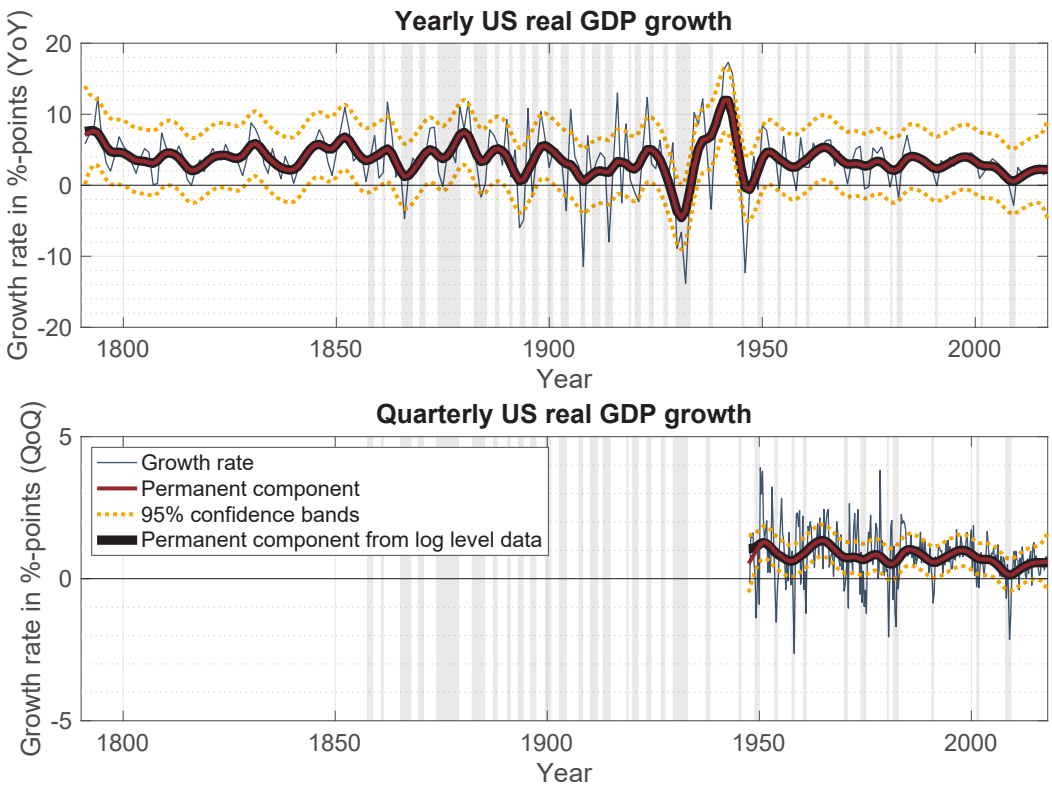
In this subsection, I apply the two-sided HP filter to derive permanent components of output growth. I also calculate the 95% confidence band for the estimated permanent components by applying Giles (2012) method. I use growth rates rather than (log) level data, as is common in the literature when applying the HP filter. The reason for using growth rates rather than (log) level data is twofold. First, growth rates are useful given the objective of this section, that is, to investigate the slope of the growth path. Therefore, the estimated permanent component directly refers to the slope of the output growth path. Second, as shown in Giles (2012), to calculate confidence bands for the permanent component of the HP filtered time series, the series needs to be stationary.[9] However, the difference in the HP trend with growth rates and the first difference of the HP trend with log level data are very small, i.e., the sample's start and end exhibit bigger differences, while in the middle, the differences are almost zero. This finding points to very similar results for the permanent component estimates.

The crucial parameter λ is $\lambda = 1600$ for quarterly data and $\lambda = 6.25$ for yearly data, as is common in the literature.[10] Note that as λ decreases, the permanent component becomes smoother. In addition, as λ decreases, the confidence bands widen.

Figure 5 plots the actual output growth rate and its permanent component for yearly and quarterly data when applying the HP filter. The average quarterly permanent component estimate is +0.77% p.q. (+3.13% p.a., note the time interval is Q1-1947–Q4-2017), while that for the average for yearly data equals +3.65% p.a. (time interval 1790–2017). The permanent component of growth rates is relatively stable, and there are rarely significantly different estimates. For example, the derived permanent component growth rate (i.e., slope) during the Great Recession (2008–2009) for yearly data is approximately +1.00%. This estimate is not significantly different for the whole period prior to 1914 or the post-war period. In addition, note that estimates are almost never significantly different from zero,

[9] See Appendix A for the form of the filter and the calculation of confidence bands.
[10] See Ravn & Uhlig (2002).

Figure 5: HP filtered permanent component of real GDP growth rates (1790–2017)



The figure plots the growth rate (blue line) and its permanent component (trend component, red line) derived when applying the two-sided HP filter with $\lambda = 1600$ for quarterly data and $\lambda = 6.25$ for yearly data. The dotted orange lines show the 95% confidence bands for the estimated permanent component. The thick black line is the trend component derived when using log level data and then first differentiating the trend component. The grey areas indicate the NBER recessions.

which is particularly true for the post-war period. There is only one big shift, which happened during the Great Depression and the Second World War.

Confidence bands widen at the beginning and end of the sample periods because I apply the two-sided filter, which features a well-known problem at the sample end points. These points are characterized by spurious dynamics.[11]

As a robustness check, I applied the same procedure to yearly data from 1950 to 2017 and obtained similar results. A significantly different trend estimate is almost impossible to find. However, confidence bands become tighter.

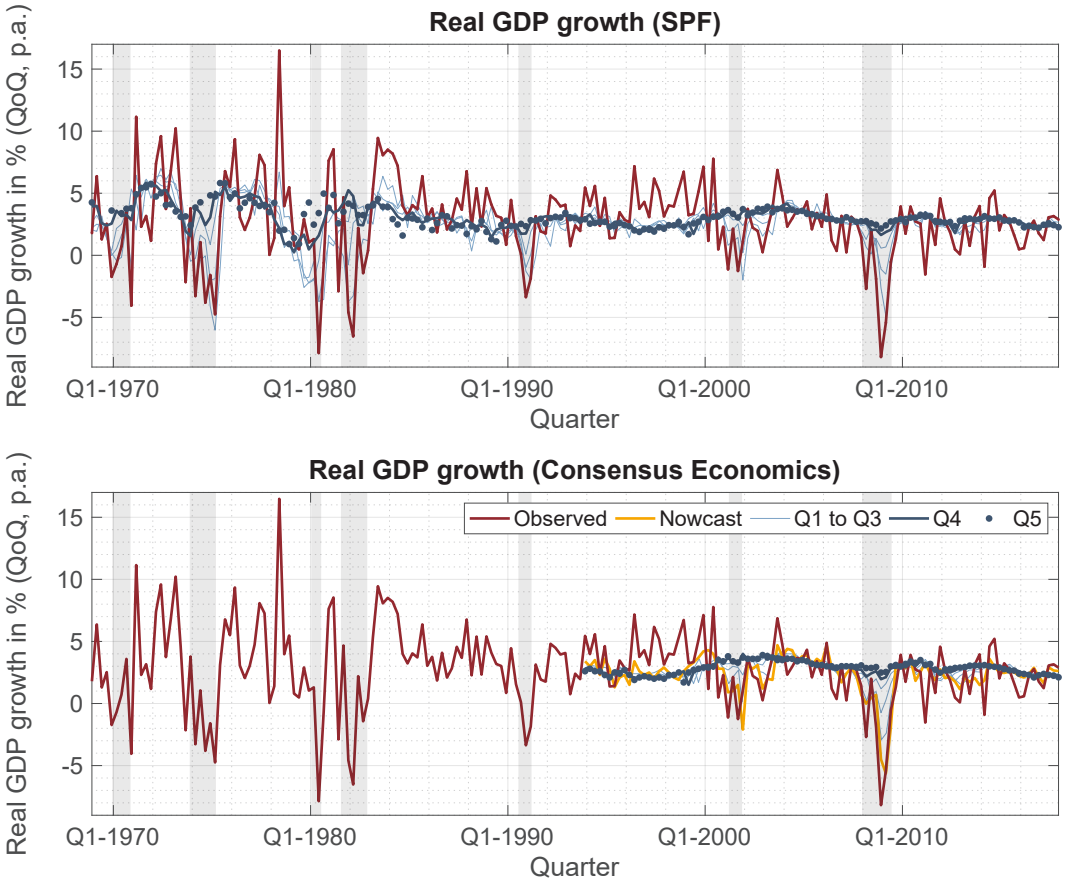
[11] See, for instance, Hamilton (2017).

To sum up, it is difficult to find permanent component estimates that are significantly different compared to their relative past estimates when applying the HP filter, which supports the conclusion that there has been no significant permanent change in average output growth rates in recent years.

5.2 Survey forecasts

In this subsection, I use survey forecasts as estimates of the permanent component. This approach is in the spirit of Campbell & Mankiw (1987a,b, 1989). If a permanent change occurs, forecasts should change significantly.

Figure 6: Term structure of cross-sectional mean forecasts (1968–2017)



The figure plots the term structure of cross-sectional mean forecasts of US real GDP growth (nowcast to forecast horizon of 5 quarters), including the observed value at time t. Quarterly data from Q4-1968 to Q4-2017 are taken from the *Survey of Professional Forecasters (SPF)* collected by the *Federal Reserve Bank of Philadelphia* (upper panel) and from Q4-1993 to Q4-2017 from the Survey by *Consensus Economics* (lower panel), with annualized quarter-on-quarter percentage change. Grey shaded areas are NBER recessions.

Figure 6 plots the term structure of cross-sectional mean forecasts for real GDP growth from the late 1960s to 2017. The frequency is quarterly. Survey forecasts are taken from the *Survey of Professional Forecasters* (*SPF*, upper panel) compiled by the Federal Reserve Bank of Philadelphia and the survey by *Consensus Economics* (lower panel). *SPF* begins in Q4-1968, while *Consensus Economics* begins in Q4-1993. Both surveys collect real GDP growth forecasts from professionals.

As **Figure 6** reveals, as the forecast horizon increases, the forecasts are less volatile. The four-quarter-ahead forecast already looks rather stable, which indicates that real GDP growth is stationary and that fluctuations in real GDP growth are dominated by transitory deviations.[12] Thus, it should be sufficient to use the longest forecast horizon (four quarters) available to estimate the transitory and permanent components. Participants in both surveys do not report their confidence about the forecasts. Thus, confidence bands are not available for either individual forecasts or cross-sectional mean forecasts. However, I use the cross-sectional standard deviation – dispersion – as a proxy for the standard deviation used to calculate confidence bands. This variable is a proxy for a narrow bound of confidence.

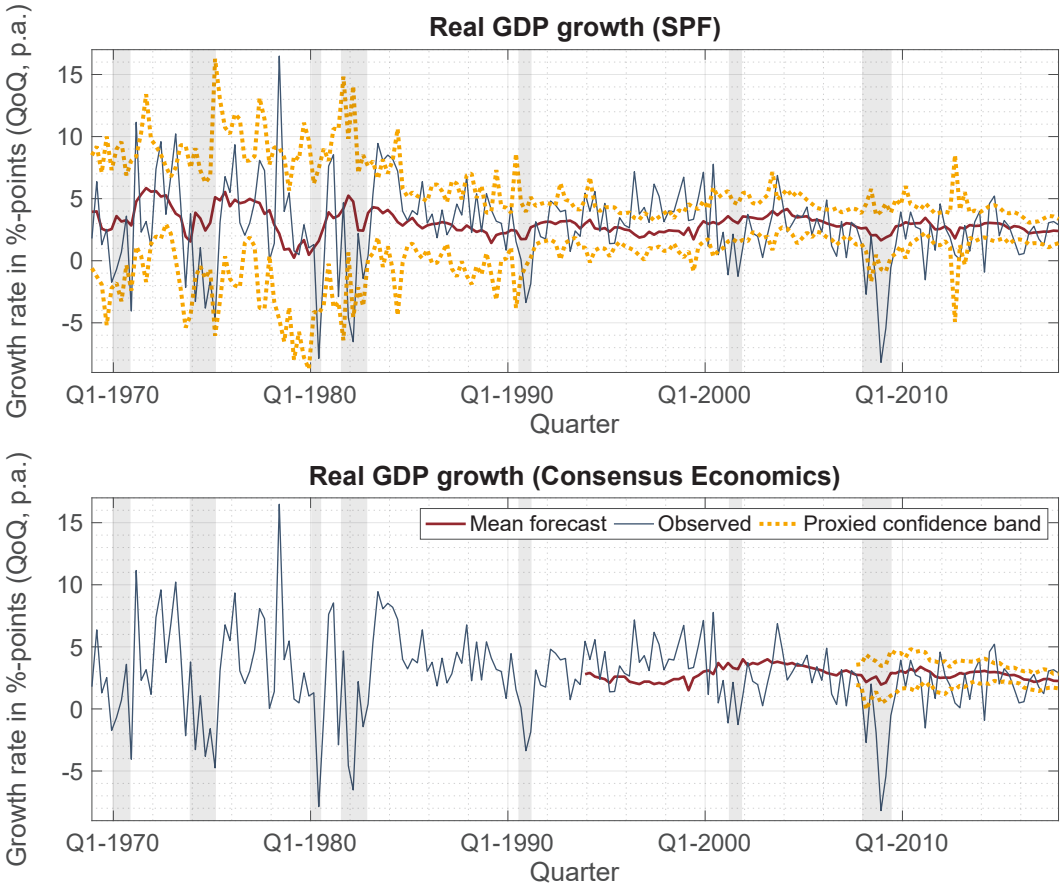
There is a caveat, however. The presented procedure assumes that forecasters report their forecast honestly, which means that they do not follow any strategic behaviour when reporting their forecasts.

Figure 7 presents the cross-sectional mean forecasts, their proxied confidence bands derived when using the cross-sectional standard deviation of reported forecasts, and the observed growth rates. The upper panel presents *SPF* observations, while the lower panel shows *Consensus Economics* forecasts. The cross-sectional standard deviation in the *Consensus Economics* survey is available from Q3-2007 onwards. The figure reveals that there is almost no permanent component estimate at a certain point in time that is outside the proxied confidence band of all other estimates, which indicates that there is no permanent change in the slope of the output growth path. Note that the proxied confidence band for *Consensus Economics* is smaller than that for the *SPF*.

[12] See, for instance, Campbell & Mankiw (1987b) and Hamilton (2017) for more details about this argument.

To summarize, the survey forecasts appear to be relatively stable and do not reveal any significant change in the slope of the output growth path.

Figure 7: Longer-term forecasts and cross-sectional forecast dispersion (1968–2017)



The figure plots the mean forecast for US real GDP growth with the longest possible forecast horizon in the survey. Quarterly data from Q4-1968 to Q4-2017 are taken from the *Survey of Professional Forecasters (SPF)* collected by the *Federal Reserve Bank of Philadelphia* (upper panel). Their forecast horizon is 5 quarters ahead. From Q4-1993 to Q4-2017, the Survey by *Consensus Economics* is plotted in the lower panel. Their forecast horizon is 4 quarters ahead. Data refer to quarter-on-quarter percentage change, annualized. The orange lines are proxies for confidence bands, where I use the cross-sectional standard deviation of forecasts (dispersion) at a certain date to calculate confidence bands. The grey shaded areas indicate NBER recessions.

6 Estimating level shifts: A bivariate time series model

In this section, I apply the procedure by Blanchard & Quah (1989) – a bivariate time series model – to estimate level shifts of the output growth path. In **Subsection 6.1**, I explain the idea of Blanchard & Quah (1989). In **Subsection 6.2**, I apply the procedure to yearly data from 1869 to 2017 and discuss their impulse

response functions. I also discuss the structural disturbances – one of them is associated with level shifts.

A permanent change in the Blanchard & Quah (1989) model differs from the univariate cases in **Section 4** and **Section 5**. In particular, in the univariate cases, I test whether the average growth rate changed over time (slope changes), while in the following, a permanent change is interpreted as a level shift of real GDP. **Figure 1** explains this distinction in a stylized manner. A key assumption of this model is that output growth rates are stationary around a certain level, i.e., there is no significant change in the slope of the output growth path.[13]

6.1 The Blanchard & Quah model

In this subsection, I present the VAR (Vector Autoregression) by Blanchard & Quah (1989) and their long-run restriction, which allows the structural disturbances in the economy to be identified. Blanchard & Quah (1989) defined a reduced-form VAR of the form[14]

$$X_t = \left(A_1 L + A_2 L^2 + \cdots + A_p L^p\right) X_t + \varepsilon_t \tag{1}$$

with

$$X_t = \begin{bmatrix} \Delta y_t \\ U_t \end{bmatrix} \sim I(0) \quad , \quad \varepsilon_t = \begin{bmatrix} \varepsilon_t^{\Delta y} \\ \varepsilon_t^{U} \end{bmatrix}$$

$X_t \sim I(0)$ says that X_t is integrated of order zero. Furthermore, L is the lag operator. ε_t is white noise, i.e., $\varepsilon_t \sim (0, \Sigma_\varepsilon)$. $\Delta y_t = \log[y_t] - \log[y_{t-1}]$. y_t is the level of real GNP and U_t is the unemployment rate, both at time period t. A_1 to A_p are 2×2 matrices since there are two variables in the system – output growth Δy_t and the unemployment rate U_t. p is the number of lags in the VAR system.

Blanchard & Quah (1989) use quarterly observations from Q2-1948 to Q4-1987. They chose $p = 8$, which corresponds to two years (8 quarters). They demean their quarterly real GNP growth rates using a structural break in Q4-1973. Similarly, they remove a fitted trend line from the unemployment rate. They justify these adjustments to their data with the assumption that in their theoretical

[13] See, for instance, Kilian & Lütkepohl (2017) for a detailed discussion.
[14] I follow Kilian & Lütkepohl (2017) closely and use their notation. Kilian & Lütkepohl (2017) present the method by Blanchard & Quah (1989) in Chapter 10.

model, both the output growth rate and the unemployment rate are stationary around a given level, i.e., $X_t \sim I(0)$. However, they found that post-war data for the US exhibit a small but steady increase in the average unemployment rate over their sample and a decline in the average output growth rate, which began in the 1970s.

Rearranging **Equation (1)** leads to

$$\left(I_2 - A_1 L - A_2 L^2 - \ldots - A_p L^p\right) X_t = A(L) X_t = \varepsilon_t$$

with I_2 as the identity matrix of size 2×2. Although the reduced-form VAR in **Equation (1)** can be estimated, a structural form is needed in order to obtain an interpretation of the parameters. Blanchard & Quah (1989) define the structural form of the economy as[15]

$$B(L) X_t = \eta_t$$

with $B(L) = B_0 - B_1 L - \cdots - B_p L^p$. The normalization $\eta_t = \left(\eta_t^S, \eta_t^D\right)' \sim (0, I_2)$ is imposed. η_t are the structural disturbances. In the model by Blanchard & Quah (1989), η_t^D are demand disturbances, which are temporary. η_t^S are supply disturbances, which are permanent. Demand (η_t^D) and supply (η_t^S) disturbances are uncorrelated with each other. Both kinds of disturbances have no long-run effects on unemployment. However, supply disturbances (η_t^S) have a long-run effect on output, while demand disturbances (η_t^D) have no long-run effect on output.

Blanchard & Quah (1989) show a simple theoretical model from which their interpretation of the disturbances is derived. Their interpretation of disturbances is based on a Keynesian view of fluctuations. On one hand, demand disturbances have short-run effects on output and unemployment due to nominal rigidities. These effects disappear over time, however. On the other hand, supply disturbances affect output in the long-run.

[15] Notation is as in Kilian & Lütkepohl (2017).

Note that $B(L) = B_0 A(L)$ and $B(1) = B_0 A(1)$. Kilian & Lütkepohl (2017) state that

$$\eta_t = B_0 \varepsilon_t \tag{2}$$

Therefore, $\Sigma_\varepsilon = B_0^{-1} B_0^{-1\prime}$. The structural moving average (MA) representation is then written as

$$X_t = B(L)^{-1} \eta_t = \Theta(L) \eta_t$$

Kilian & Lütkepohl (2017) write that any structural disturbance of X_t will fade away as the horizon increases because X_t is $I(0)$. By construction, output growth Δy_t and the unemployment rate U_t will return to their initial values. However, the level of real GNP will not necessarily return to its initial (trend growth) value. The long-run effect equals $\Theta(1) = \sum_{i=0}^{\infty} \Theta_i = B(1)^{-1}$, which is illustrated in **Figure 1** (lower right panel). If a supply disturbance (permanent disturbance) occurs, a level shift in output occurs.

To derive such a result, Blanchard & Quah (1989) impose a long-run restriction on the effect of demand disturbances on output growth. According to Kilian & Lütkepohl (2017), this situation leads to an exclusion restriction (upper-left value is 0), which is written as

$$\Theta(1) = \begin{bmatrix} \theta_{11}(1) & 0 \\ \theta_{21}(1) & \theta_{22}(1) \end{bmatrix}$$

Kilian & Lütkepohl (2017) explain that the exclusion restriction guarantees that demand disturbances do not affect the level of output (y_t) in the long run. The first element of $\Theta(1)$ stays unrestricted because aggregate supply disturbances affect the level of output in the long run. The second row of $\Theta(1)$ is unrestricted because the cumulative responses of the unemployment rate U_t are different from zero. This is because U_t is stationary. A cumulative response on a stationary variable is obviously not 0. According to Kilian & Lütkepohl (2017), we can write

$$A(1)^{-1} \Sigma_u \left[A(1)^{-1} \right]' = \Theta(1) \Theta(1)'$$

and apply a lower triangular Cholesky decomposition to $A(1)^{-1} \Sigma_u \left[A(1)^{-1} \right]'$ to

derive $\Theta(1)$. From this, we can calculate

$$B_0^{-1} = A(1)\Theta(1)$$

which allows us to estimate the structural disturbances η_t when applying **Equation (2)**.

To summarize, using the above assumptions, it is possible to distinguish the structural disturbances in the economy. This approach allows the level shifts in the output growth path to be estimated.

6.2 Estimating the Blanchard & Quah model

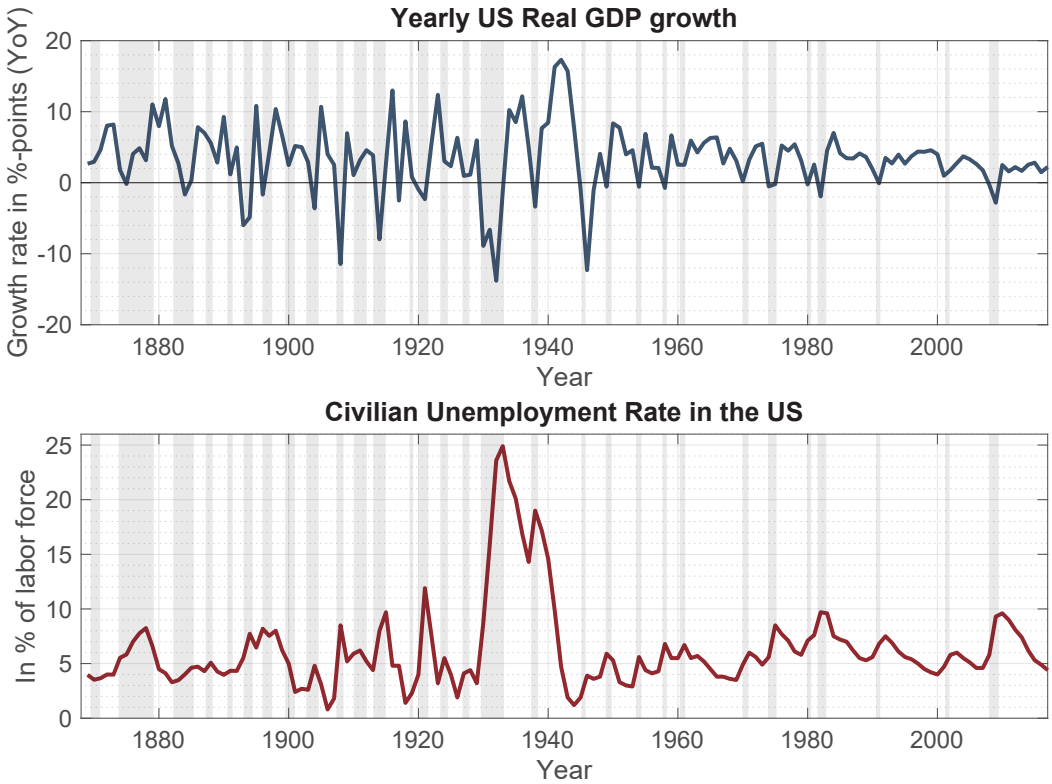
In this section, I estimate the reduced VAR by Blanchard & Quah (1989), calculate impulse response functions and identify the structural disturbances in the economy.

In contrast to Blanchard & Quah (1989), I use yearly real GDP starting in 1869. Output growth data (yearly GDP growth rates) are described in **Section 3**. Sources for the unemployment rate are as follows: For the unemployment rate from 1869 to 1899, I use estimates by Vernon (1994). From 1900 to 1947, I use the unemployment rate provided by the Bureau of the Census (1960).[16] From 1948 to 2017, data are downloaded from FRED.[17] **Figure 8** presents the data. Peculiarly, the spike in the unemployment rate during the Great Depression (1929–1933) is impressive. Unemployment is almost double that experienced before and after the Great Depression.

When estimating their model, Blanchard & Quah (1989) assume that output growth rates, as well as the unemployment rate, are stationary around a certain level. Augmented Dickey-Fuller tests reveal that real GDP growth and the unemployment rate do not exhibit a unit root. The tests were executed with a 1 lag and an intercept. Test values are -7.35 and -4.18, respectively. The corresponding critical value is -2.88. Thus, the null hypothesis of a unit root can be rejected. The assumption $X_t \sim I(0)$ (stationarity) is thus satisfied.

[16] See Bureau of the Census (1960), Table D 46-47. Unemployment: 1900 to 1957, p. 73.
[17] FRED Ticker UNRATE, called *Civilian Unemployment Rate, Percent, Annual, Seasonally Adjusted*, downloaded in June 2018.

Figure 8: Yearly output growth and unemployment rate (1869–2017)

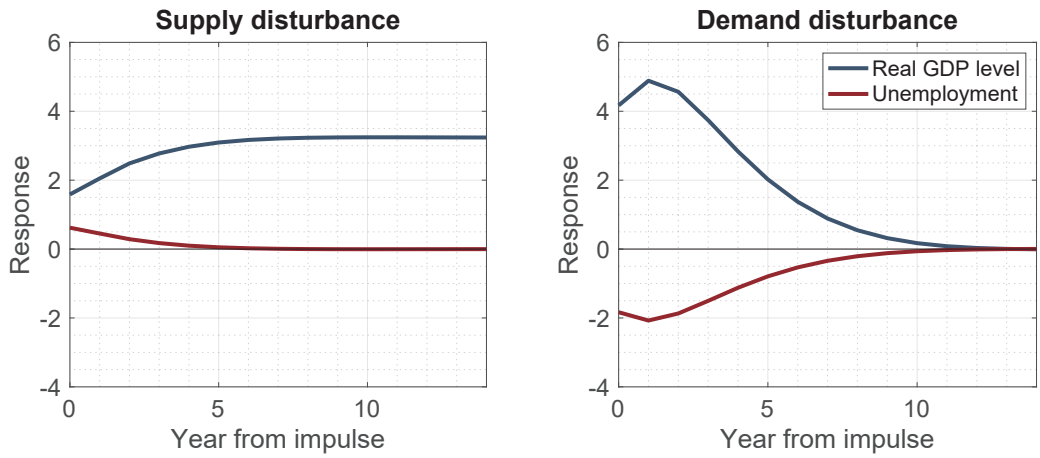


The figure plots the yearly US real GDP growth rate and the average yearly civilian unemployment rate in the US from 1869 to 2017. Grey shaded areas indicate NBER recessions.

Furthermore, in **Section 4** and **Section 5**, there is no evidence of changes in average output growth rates (slope change). Christiano (1992) also casts some doubt about a structural break in post-war output. Therefore, I refrain from further structural break tests for output growth. In addition, a regression of the variable in question on a constant and a trend revealed that real GDP growth and unemployment do not exhibit a trend. Hence, in contrast to Blanchard & Quah (1989), I simply demean both time series.

Blanchard & Quah (1989) use up to 8 lags in their estimation when using quarterly data. Consequently, I use $p = 2$ lags since the data are at a yearly frequency.

Figure 9: Impulse-response of a one-standard-deviation disturbance (1869–2017)



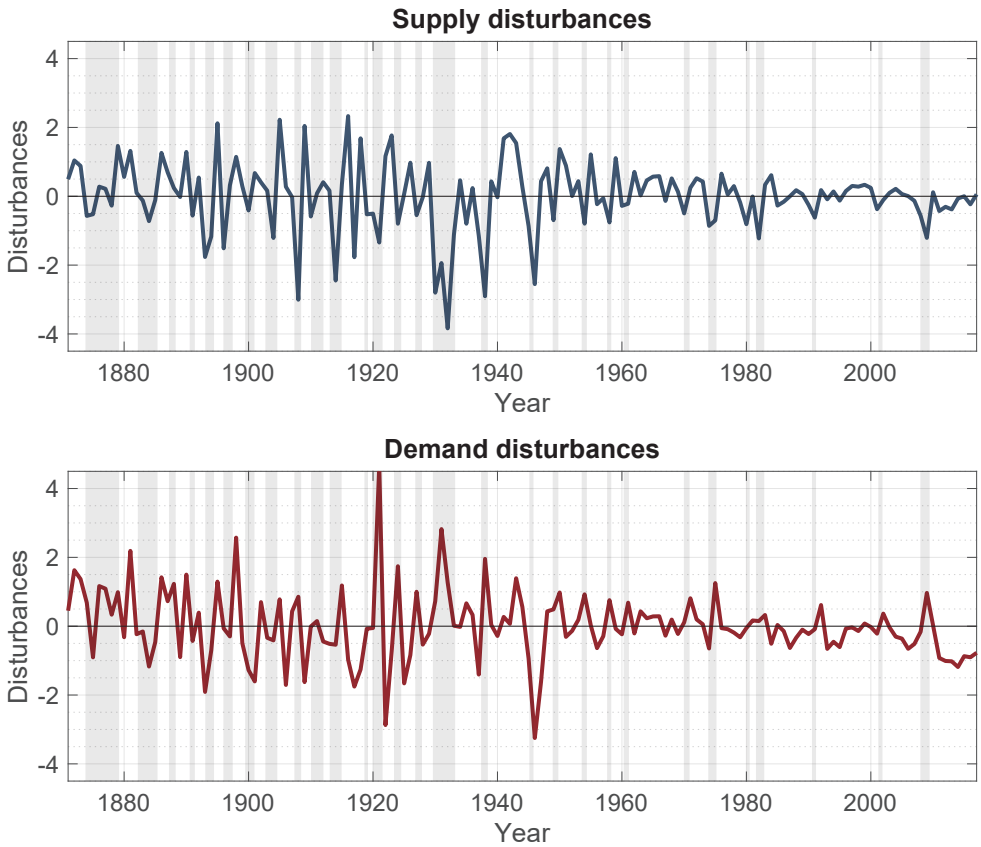
The figure plots the impulse-response function of a one-standard-deviation disturbance in supply (left panel) and in demand (right panel) for the following 15 years (x-axis). The y-axes denote simultaneously the log of output and the rate of unemployment.

Figure 9 plots the impulse response function (IRF) of a supply and demand disturbance when estimating the model by Blanchard & Quah (1989).[18] The vertical axes simultaneously denote the rate of unemployment and the log of real GDP. The observed patterns are very similar to Blanchard & Quah (1989). However, generally speaking, the impulse-response is less dynamic for yearly data compared to quarterly data estimates by Blanchard & Quah (1989).

Supply disturbances have a permanent effect on the level of output. In contrast to that found by Blanchard & Quah (1989), the effect is not hump-shaped but cumulates steadily over time. After approximately five years, the effect is almost double the initial value and already stabilized. Supply disturbances do not affect unemployment permanently, and after four years, the initial disturbance vanishes. Demand disturbances affect neither the GDP level nor the unemployment rate permanently, with the initial disturbances fading away after ten years. Like Blanchard & Quah (1989), demand disturbances have a hump-shaped effect on the level of output and an inverse hump-shaped effect on unemployment.

[18] I used the Matlab Toolbox by Ambrogio Cesa-Bianchi to estimate the reduced-form VAR and the corresponding IRFs. I downloaded the Toolbox in September 2017, `https://sites.google.com/site/ambropo/home`.

Figure 10: Structural demand and supply disturbances (1871–2017)



The figure plots the estimated structural demand and supply disturbances from 1871 to 2016. Both disturbances have a variance of 1 and are uncorrelated with each other by construction. The grey shaded areas are the NBER recessions.

A permanent change in the level of output is associated with structural supply disturbances η_t^S. **Equation (2)** gives the relationship between the reduced-form VAR innovations ε_t and the structural disturbances η_t, which are white noise, i.e., $\eta_t \sim (0, I_2)$. Given **Equation (2)**, we know $\Sigma_\varepsilon = B_0^{-1} B_0^{-1\prime}$. Moreover, I can estimate Σ_ε since I can estimate ε_t from the reduced-form VAR and, thus, I also know $A(1)$. As described in **Subsection 6.1**, imposing a lower triangular Cholesky decomposition on $A(1)^{-1}\Sigma_u \left[A(1)^{-1}\right]'$ leads to an estimate of B_0, and therefore, $\eta_t = B_0 \varepsilon_t$.

Figure 10 presents supply and demand disturbances. The figure illustrates that both supply and demand disturbances were more volatile before 1947, which means that large permanent shifts were more common during this period than during the post-war period. Obviously, permanent shifts in the level of real GDP were less pronounced in the post-war period. In addition, there was no

large permanent shift during the financial crisis or during the 1970s (oil crisis) compared to shifts in the pre-war period.

To sum up, the effect of permanent disturbances on output is similar to that in Blanchard & Quah (1989). The results show that there were only small permanent disturbances in the post-war period and that both permanent and transitory disturbances were more volatile in the pre-war period. Gordon's "special century" thus exhibited more volatile permanent level shifts. However, this finding does not indicate a permanent slope change, as he predicts.

7 Conclusions

In this paper, I derived and applied three univariate methods and one bivariate method to estimate permanent and transitory components of the US output growth path over a long period of time. The results revealed that there is little support for the hypothesis of significant changes in the slope of the output growth path. The bivariate method showed that permanent shifts in the output growth path were relatively small in the post-war period. In contrast, Gordon's "special century" exhibited more volatile permanent level shifts. Overall, it is difficult to find permanent changes in the output growth path that are statistically significant.

Future research may elaborate on more sophisticated models. For example, unobserved component models may lead to more nuanced conclusions. It would also be particularly interesting to determine whether the observed productivity decline and the decrease in r^* estimates in recent years are statistically significant. Adding confidence bands to such estimates is often ignored in the literature and public debates. As this paper shows, reporting point estimates with confidence bands may weaken conclusions derived from point estimates that do not consider confidence bands.

References

Beveridge, S. & Nelson, C. R. (1981), 'A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'', *Journal of Monetary Economics* **7**, 151–174.

Blanchard, O. J. & Quah, D. (1989), 'The Dynamic Effects of Aggregate Demand and Supply Disturbances', *The American Economic Review* **79**(4), 655–673.

Blanchard, O. J. & Quah, D. (1993), 'The Dynamic Effects of Aggregate Demand and Supply Disturbances: Reply', *The American Economic Review* **83**(3), 653–658.

Bureau of the Census (1960), *U.S. Bureau of the Census, Historical Statistics of the United States, Colonial Times to 1957*, library of congress card no. a 60-9150 edn, Bureau of the Census, Washington D.C.

Campbell, J. Y. & Mankiw, G. N. (1987a), 'Permanent and Transitory Components in Macroeconomic Fluctuations', *The American Economic Review: Papers & Proceedings* **77**(2), 111–117.

Campbell, J. Y. & Mankiw, G. N. (1987b), 'Are Output Fluctuations Transitory?', *The Quarterly Journal of Economics* **102**(4), 857–880.

Campbell, J. Y. & Mankiw, G. N. (1989), 'International Evidence on the Persistence of Economic Fluctuations', *Journal of Monetary Economics* **23**, 319–333.

Christiano, L. J. (1992), 'Searching for a Break in GNP', *Journal of Business & Economic Statistics* **10**(3), 237–250.

Cochrane, J. H. (1988), 'How Big Is the Random Walk in GNP?', *Journal of Political Economy* **96**(5), 893–920.

Cochrane, J. H. (1994), 'Permanent and Transitory Components of GNP and Stock Prices', *The Quarterly Journal of Economics* **109**(1), 241–265.

Coibion, O., Gorodnichenko, Y. & Ulate, M. (2017), 'The Cyclical Sensitivity in Estimates of Potential Output', *NBER Working Paper Series* (23580).

Danthine, J.-P. & Girardin, M. (1989), 'Business Cycles in Switzerland', *European Economic Review* **33**, 31–50.

Giles, D. E. (2012), 'Constructing Confidence Bands for the Hodrick-Prescott Filter', *Econometrics Working Paper EWP* (1202).

Gordon, R. J. (2015), 'Secular Stagnation: A Supply-Side View', *American Economic Review: Papers & Proceedings* **105**(5), 54–59.

Gordon, R. J. (2016), *The Rise and Fall of American Growth*, Princeton University Press.

Hamilton, J. D. (2017), 'Why You Should Never Use the Hodrick-Prescott Filter', *NBER Working Paper Series* (23429).

Hodrick, R. J. & Prescott, E. C. (1997), 'Postwar U.S. Business Cycles: An Empirical Investigation', *Journal of Money, Credit and Banking* **29**(1), 1–16.

Keating, J. W. & Nye, J. V. (1998), 'Permanent and Transitory Shocks in Real Output: Estimates from Nineteenth-Century and Postwar Economies', *Journal of Money, Credit and Banking* **30**(2), 231–251.

Kilian, L. & Lütkepohl, H. (2017), *Structural Vector Autoregressive Analysis*, Cambridge University Press.

Lippi, M. & Reichlin, L. (1993), 'The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment', *The American Economic Review* **83**(3), 644–652.

Nelson, C. R. & Plosser, C. I. (1982), 'Trends and Random Walks in Macroeconomic Time Series', *Journal of Monetary Economics* **10**, 139–162.

Ravn, M. O. & Uhlig, H. (2002), 'Notes: On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations', *The Review of Economics and Statistics* **84**(2), 371–380.

Vernon, J. R. (1994), 'Unemployment Rates in Postbellum America: 1896-1999', *Journal of Macroeconomics* **16**(4), 701–714.

Appendix A HP filter and confidence bands

In this section, I show how to derive confidence bands for the HP filter trend component according to Giles (2012).[19] The goal of the HP filter is to separate a time series x_t into a cyclical component c_t (transitory component) and a trend component g_t (permanent component), that is,

$$x_t = g_t + c_t \tag{3}$$

According to Hodrick & Prescott (1997), the HP filter is written as

$$\begin{aligned} \min_{\{g_t\}_{t=1}^N} \quad & \sum_{t=1}^{N} c_t^2 + \lambda \cdot \sum_{t=3}^{N} \left[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2}) \right]^2 \\ & \text{s.t. } c_t = x_t - g_t \end{aligned} \tag{4}$$

λ is the parameter that penalizes variations in growth rates of the trend component. The larger the λ, the higher the penalty. N is the number of observations. According to Danthine & Girardin (1989), the HP filter in matrix notation is written as

$$\begin{aligned} \min_{\{g_t\}_{t=1}^N} \quad & c_t' c_t + \lambda \cdot (Kg_t)' (Kg_t) \\ & \text{with } Kg_t = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_N \end{bmatrix} \end{aligned}$$

Danthine & Girardin (1989) show that the solution to this problem equals

$$g_t^* = \left(I + \lambda K' K \right)^{-1} x_t \tag{5}$$

where I is the identity matrix of order N, which leads to the covariance matrix

$$\mathbb{V}\left[g_t^* \right] = \left(I + \lambda K' K \right)^{-1} \mathbb{V}\left[x_t \right] \left(I + \lambda K' K \right)^{-1} \tag{6}$$

$\mathbb{V}\left[g_t^* \right]$ can be used to construct confidence bands for g_t^* (permanent component). The crucial point is how to estimate $\mathbb{V}\left[x_t \right]$, which depends on the underlying

[19] In this presentation, I closely follow Dave Giles's Econometrics blog, see `http://davegiles.blogspot.ch/2011/12/confidence-bands-for-hodrick-prescott.html`, viewed in October 2017.

time series x_t. I assume growth rates follow a stationary AR(1)

$$x_t = \mu + \phi \cdot x_{t-1} + \varepsilon_t$$

with $|\phi| < 1$ and $\varepsilon_t \sim N(0, \sigma^2)$.[20] Thus,

$$\mathbb{V}\left[g_t\right] = \left(I + \lambda K'K\right)^{-1} \frac{\sigma^2}{1-\phi^2} \begin{bmatrix} 1 & \phi^1 & \phi^2 & \cdots & \phi^N \\ \phi^1 & 1 & \phi^1 & \cdots & \phi^{N-1} \\ \phi^2 & \phi^1 & 1 & \cdots & \phi^{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi^N & \phi^{N-1} & \phi^{N-2} & \cdots & 1 \end{bmatrix} \left(I + \lambda K'K\right)^{-1}$$

To construct confidence intervals, I estimate an AR(1). I use the estimates of ϕ and σ^2 and the diagonal of $\mathbb{V}\left[g_t\right]$ to construct 95% confidence bands.

[20] Estimated sample autocorrelation and partial autocorrelation of $x_t = \Delta y_t \cdot 100$ support this assumption. Both measures fade away after one lag for quarterly and yearly data.

Recent SNB Working Papers

2018-14 Thomas Lustenberger:
Has the American Output Growth Path Experienced a Permanent Change?

2018-13 Stephan Imhof, Cyril Monnet and Shengxing Zhang:
The Risk-Taking Channel of Liquidity Regulations and Monetary Policy

2018-12 Andreas M. Fischer, Henrike Groeger, Philip Sauré and Pinar Yeşin:
Current account adjustment and retained earnings

2018-11 Maxime Phillot, Rina Rosenblatt-Wisch:
Inflation Expectations: The Effect of Question Ordering on Forecast Inconsistencies

2018-10 Alex Cukierman, Thomas Lustenberger:
International Evidence on Professional Interest Rate Forecasts:
The Impact of Forecasting Ability

2018-9 Thomas Nitschka:
Did China's anti-corruption campaign affect the risk premium on stocks of global luxury goods firms?

2018-8 Andreas M. Fischer, Lucca Zachmann:
Do the rich pay their taxes early?

2018-7 Basil Guggenheim, Mario Meichle and Thomas Nellen:
Confederation debt management since 1970

2018-6 Lucas Marc Fuhrer, Basil Guggenheim and Matthias Jüttner:
What do Swiss franc Libor futures really tell us?

2018-5 Barthélémy Bonadio, Andreas M. Fischer and Philip Sauré:
The speed of exchange rate pass-through

2018-4 Simone Auer, Christian Friedrich, Maja Ganarin, Teodora Paligorova, and Pascal Towbin:
International Monetary Policy Transmission trough Banks in Small Open Economies

2018-3 Romain Baeriswyl, Camille Cornand and Bruno Ziliotto:
Observing and shaping the market: the dilemma of central banks

2018-2 Adriel Jost:
Cultural Differences in Monetary Policy Preferences

2018-1 Reto Foellmi, Adrian Jaeggi and Rina Rosenblatt-Wisch:
Loss Aversion at the Aggregate Level Across Countries and its Relation to Economic Fundamentals

2017-18 Gregor Bäurle, Sarah M. Lein and Elizabeth Steiner:
Employment Adjustment and Financial Constraints – Evidence from Firm-level Data

2017-17 Thomas Lustenberger and Enzo Rossi:
The Social Value of Information:
A Test of a Beauty and Non-Beauty Contest

2017-16 Aleksander Berentsen and Benjamin Müller:
A Tale of Fire-Sales and Liquidity Hoarding

2017-15 Adriel Jost:
Is Monetary Policy Too Complex for the Public?
Evidence from the UK

2017-14 David R. Haab, Thomas Nitschka:
Predicting returns on asset markets of a small, open economy and the influence of global risks

2017-13 Mathieu Grobéty:
Government Debt and Growth: The Role of Liquidity

SCHWEIZERISCHE NATIONALBANK
BANQUE NATIONALE SUISSE
BANCA NAZIONALE SVIZZERA
BANCA NAZIUNALA SVIZRA
SWISS NATIONAL BANK